**Math 3C Summer II 2015 - Exam 1 Solutions**

**Instructions:** Put your name and PID on your blue book. No calculators or electronic devices are allowed. Turn off and put away your cell phone. You may use one side of one 8.5 × 11 sheet of notes, but no other notes, books, or resources. Make sure your solutions are clear and legible, and show all your work. Credit may not be given for unreadable or unsupported answers. Write your solutions in your blue book, keeping the questions in order, and clearly indicating which problem is being solved.

**Question 0.** (1 point) Carefully read all instructions above and make sure you have followed them all, as well as any other instructions given by your instructor during the exam.

**Question 1.** (4 points) Solve the inequality $|x - 7| < 3$. Express your answer in interval notation and on a number line.

**Solution.** If $|x - 7| < 3$ then $x - 7 < 3$ and $-(x - 7) < 3$. Hence $4 < x < 10$, so any $x$ in the interval $(4, 10)$ satisfies the inequality.

**Question 2.** Write the equations of the following lines in slope-intercept form (i.e. the form $y = mx + b$):

(a) (2 points) The line through the points $(1, 2)$ and $(3, 6)$.

(b) (2 points) The line perpendicular to the line in part (a) passing through the point $(2, 1)$.

**Solution.** (a) The slope of this line is $m = \frac{6 - 2}{3 - 1} = 2$ so in point-slope form, the line is given by the equation $(y - 2) = 2(x - 1)$. In slope-intercept form, this line is

$$y = 2x.$$

(b) A line perpendicular to the line in part (a) has slope $-1/2$. In point-slope form, a line with slope $-1/2$ through the point $(2, 1)$ is given by the equation $(y - 1) = -1/2(x - 2)$, so in slope-intercept form the line has equation

$$y = -\frac{1}{2}x + 2.$$

**Question 3.** (4 points) Find the vertex and the zeros (roots) of the parabola

$$y = x^2 + 6x + 3.$$

**Solution.** By completing the square, the parabola has equation $y = (x+3)^2 - 6$. Hence, the vertex is $(-3, -6)$. By the quadratic formula, the roots occur at $x = \frac{-6 \pm \sqrt{24}}{2} = -3 \pm \sqrt{6}$. 
**Question 4.** (4 points) Find the center and radius of the circle given by the equation
\[ x^2 + 2x + y^2 - 4y = 4. \]

**Solution.** Complete the square twice to see that the circle is given by the equation
\[ (x + 1)^2 + (y - 2)^2 = 9. \]
This circle has center \((-1, 2)\) and radius 3.

**Question 5.** (4 points) Let \(f(x) = \frac{x}{\sqrt{|x| - 4}}.\) Find the largest domain where \(f(x)\) is defined. Write your answer in interval notation.

**Solution.** The function is defined as long as \(|x| - 4 \geq 0\) or \(|x| \geq 4.\) Hence, the function is defined for any \(x \geq 4\) and any \(x \leq -4.\) The largest domain of \(f(x)\) is therefore \((-\infty, -4) \cup [4, \infty)\).

**Question 6.** (4 points) Suppose \(f(x)\) is an odd function defined on the domain \([-3, 3]\). Suppose further that on \([0, 3],\) \(f(x)\) is defined by the formula
\[ f(x) = \frac{x(x - 3)}{2}. \]
What is \(f(-1)\)?

**Solution.** Since \(f(x)\) is an odd function, therefore \(f(-1) = -f(1).\) By substituting into the given formula \(f(1) = -1.\) Hence, \(f(-1) = 1.\)

**Question 7.** (4 points) Let \(f(x) = x^2 - 3\) and \(g(x) = 2x + 2.\) Find \((f \circ g)(x).\)

**Solution.** We have \((f \circ g)(x) = f(g(x)) = f(2x + 2) = (2x + 2)^2 - 3.\)

**Question 8.** (4 points) Simplify the expression
\[ \left( \frac{x^{-3}(y^2z^{-4})^{-2}}{x^4y^3z} \right)^2 \left( \frac{xy}{z^3} \right)^2. \]
Your answer should be in the form \(x^a y^b z^c\) for some numbers \(a, b, c.\)

**Solution.** This expression simplifies to \(x^{-5}y^{-5}z^5.\)
Question 9. (8 points) This is the graph of $g(x)$, a function with domain $[1,4]$:

Let $h(x) = -\frac{1}{2}g(x + 1) + 2$. What are the domain and range of $h(x)$? Graph $h(x)$.

**Solution.** The graph of $h(x)$ is below; $h(x)$ has domain $[0,3]$ and range $[1/2,2]$.

![Graph of h(x)](image)

Question 10. (4 points) Give an example of two one-to-one functions $f(x)$ and $g(x)$ whose sum $(f + g)(x)$ is not one-to-one.

**Solution.** There were (infinitely) many possible correct answers here. One example choice is $f(x) = x$ and $g(x) = -x$, two one-to-one functions whose sum $(f + g)(x)$ is the constant function 0 and therefore not one-to-one. You could also have defined your function by a table of values, for example: $f(0) = 1$, $f(1) = 0$, $g(0) = 0$ and $g(1) = 1$. 

![Graph of h(x)](image)