Math 3C Summer II 2015 - Final Exam

Instructions: Put your name and PID on your blue book. Turn off and put away your cell phone. No calculators. You may use both sides of one 8.5 × 11 sheet of notes, but no other resources. Make sure your solutions are clear and legible, and show all your work. Credit may not be given for unreadable or unsupported answers. Write your solutions in your blue book, keeping the questions in order, and clearly indicating which problem is being solved. This exam is out of 75 points, with 4 possible extra credit points.

Question 0. (1 point) Carefully read all instructions above and make sure you have followed them all, as well as any other instructions given by your instructor during the exam.

Question 1.

(a) (4 points) Suppose $\pi \leq \theta < 2\pi$ and $\tan\theta = 2$. Find $\sin\theta$.
(b) (4 points) Compute $\sin(\tan^{-1} 2)$.

Question 2. Let $f(x) = 3 \tan^{-1}(x) - 3$.

(a) (2 points) What is the largest possible domain for $f(x)$?
(b) (4 points) What is the range of $f(x)$?
(c) (4 points) Find all roots of $f(x)$.

Question 3. (4 points) Give the equation of the line that forms an angle of $30^\circ$ with the $x$-axis and intersects the $x$-axis at $(3,0)$.

Question 4. (6 points) Let $h(x) = 3 \cdot 5^{2x}$. Recall that we can also write $h(x)$ in the form $h(x) = c \cdot B^x$ or $h(x) = c \cdot 2^{kx}$. What are $c$, $B$, and $k$?
Question 5. Compute the following

(a) (2 points) $\csc \left( -\frac{11\pi}{6} \right)$
(b) (2 points) $\tan \left( \frac{15\pi}{4} \right)$
(c) (2 points) $\sin^{-1} \left( -\frac{\sqrt{3}}{2} \right)$
(d) (2 points) $\tan^{-1} \left( -\sqrt{3} \right)$

Question 6. (4 points) Find a number $a$ such that the following system has no solutions.

\[
\begin{align*}
2x + y &= 3 \\
ax - 2y &= 4
\end{align*}
\]

Question 7. The height $h(t)$ of a thrown ball (in ft) at the time $t$ seconds is given by:

\[ h(t) = -16t^2 + v_0t + h_0 \]

where $v_0$ is the initial velocity in ft/sec and $h_0$ is the initial height in ft. Suppose you throw a ball from a height of 4 ft with an initial velocity of 8 ft/sec.

(a) (2 points) At what time does the ball reach its maximum height?
(b) (2 points) What height does the ball reach at its maximum?
(c) (2 points) At what time does the ball hit the ground?

Question 8. (4 points) Solve for $x$:

\[ \ln(x + 5) + \ln(x - 1) = 2. \]

Question 9. (6 points) Suppose $g(x)$ has the form $g(x) = a\cos(bx) + c$. If $g(x)$ has period $\frac{3\pi}{4}$ and range $[-5, 1]$, find $a$, $b$, and $c$.

Question 10. (4 points) Suppose an 8 ft long ladder is propped against a wall. If the ladder makes an angle of 74° with the ground, how high up the wall does the ladder reach?

Question 11. Determine whether the following functions are odd, even, or neither.

(a) (4 points) $f(x) = \cos(x + \pi)$
(b) (4 points) $g(x) = \sin(3\pi/2 - x)$

Question 12. (4 points) Find all horizontal and vertical asymptotes of the rational function

\[ r(x) = \frac{3x^2 + 2x - 1}{16 - 4x^2}. \]

Extra credit. (4 points all-or-nothing) Let $p(x) = x^3 - 3x^2 - 2x + 4$. Find all roots of $p(x)$. (Hint: Can you guess a root?)

Good work this quarter! If you are taking calculus next quarter, best of luck!