1a)

$$\min \quad 4x_1 + 2x_2 - 33x_3$$

s.t.

$$x_1 - 4x_2 + 3x_3 + s_1 = 12$$
$$9x_1 + 6x_2 = 15$$
$$5x_1 - 9x_2 + s_2 = -3$$
$$x_1, x_2, x_3, s_1, s_2 \geq 0$$

1b)

$$\max \quad 45x_1 + 15x_2$$

s.t.

$$4x_1 - 2x_2 + 9x_3 = 22$$
$$2x_1 - 5x_2 + x_3 + s_1 = -1$$
$$+x_1 - x_2 + s_2 = 3$$
$$x_1, x_2, x_3, s_1, s_3 \geq 0$$

1c)

Let $x_3 = -x_3'$ and $x_2 = x_2^+ - x_2^-$

$$\min \quad 2x_1 + x_2^+ - x_2^- + 4x_3'$$

s.t.

$$x_1 - x_2^+ + x_2^- + 5x_3' + s_1 = 10$$
$$9x_1 + 3x_2^+ - 3x_2^- = -6$$
$$x_1, x_2^+, x_2^-, x_3', s_1 \geq 0$$

1d) Let $x_3 = x_3^+ - x_3^-$

$$\min \quad 3x_1 - 3x_2 + 7x_3^+ - 7x_3^-$$

s.t.

$$x_1 + x_2 + 3x_3^+ - 3x_3^- + s_1 = 40$$
$$-x_1 - 9x_2 + 7x_3^+ - 7x_3^- + s_2 = -50$$
$$5x_2 + 8x_3^+ - 8x_3^- + s_3 = 70$$
$$-5x_2 - 8x_3^+ + 8x_3^- + s_4 = 70$$
$$x_1, x_2, x_3^+, x_3^-, s_1, s_2, s_3, s_4 \geq 0$$

2) max $2x_1 + 7x_2$

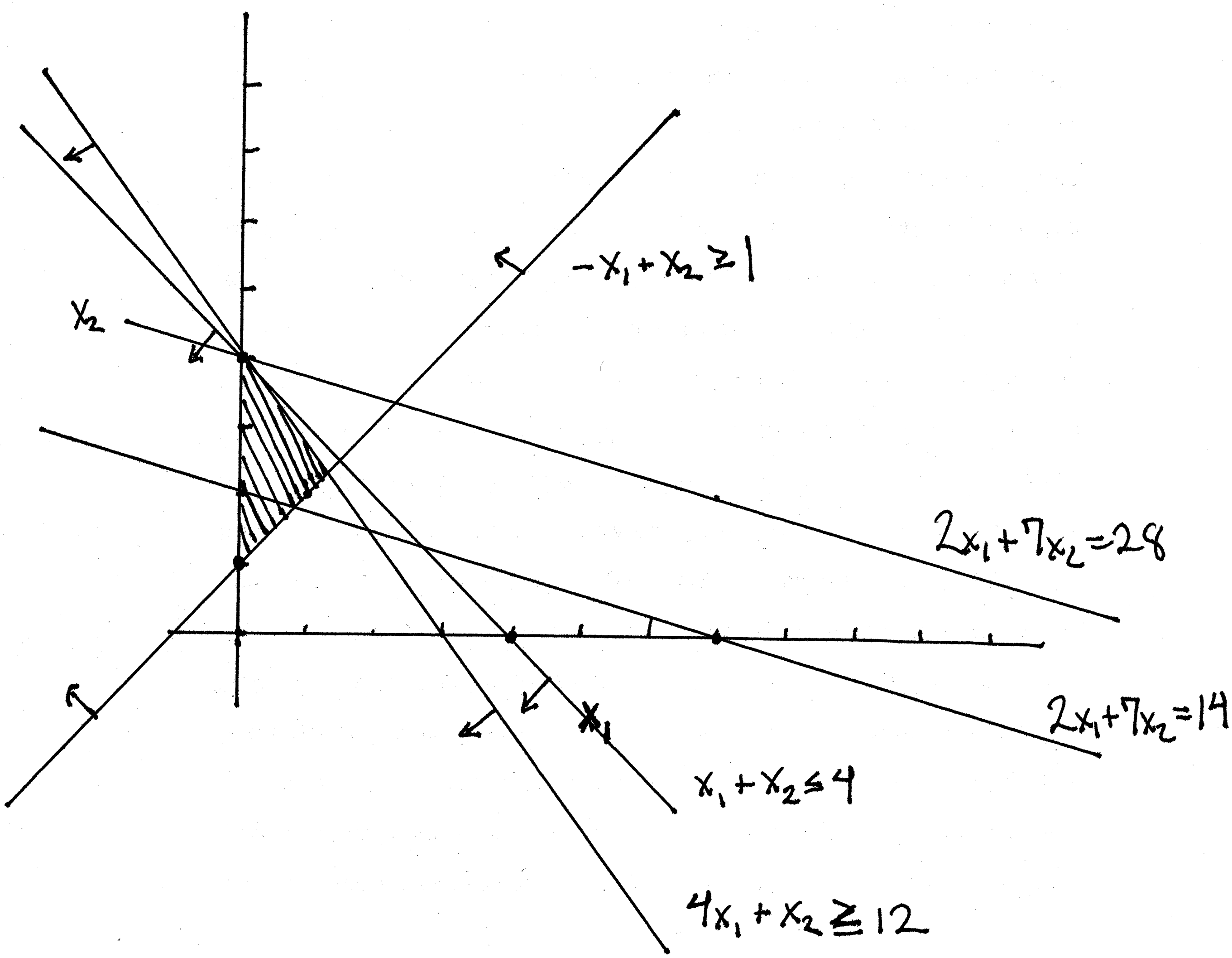
s.t.

$$x_1 + x_2 \le 4$$

$$4x_1 + 3x_2 \le 12$$

$$-x_1 + x_2 \ge 1$$

$$x_1, x_2 \ge 0$$



Optimal Solution $(x_1, x_2) = (0, 4)$ with objective value 28

3)

$$\max \; z = \min\{3x_1 - 10, -5x_1 + 5\}$$

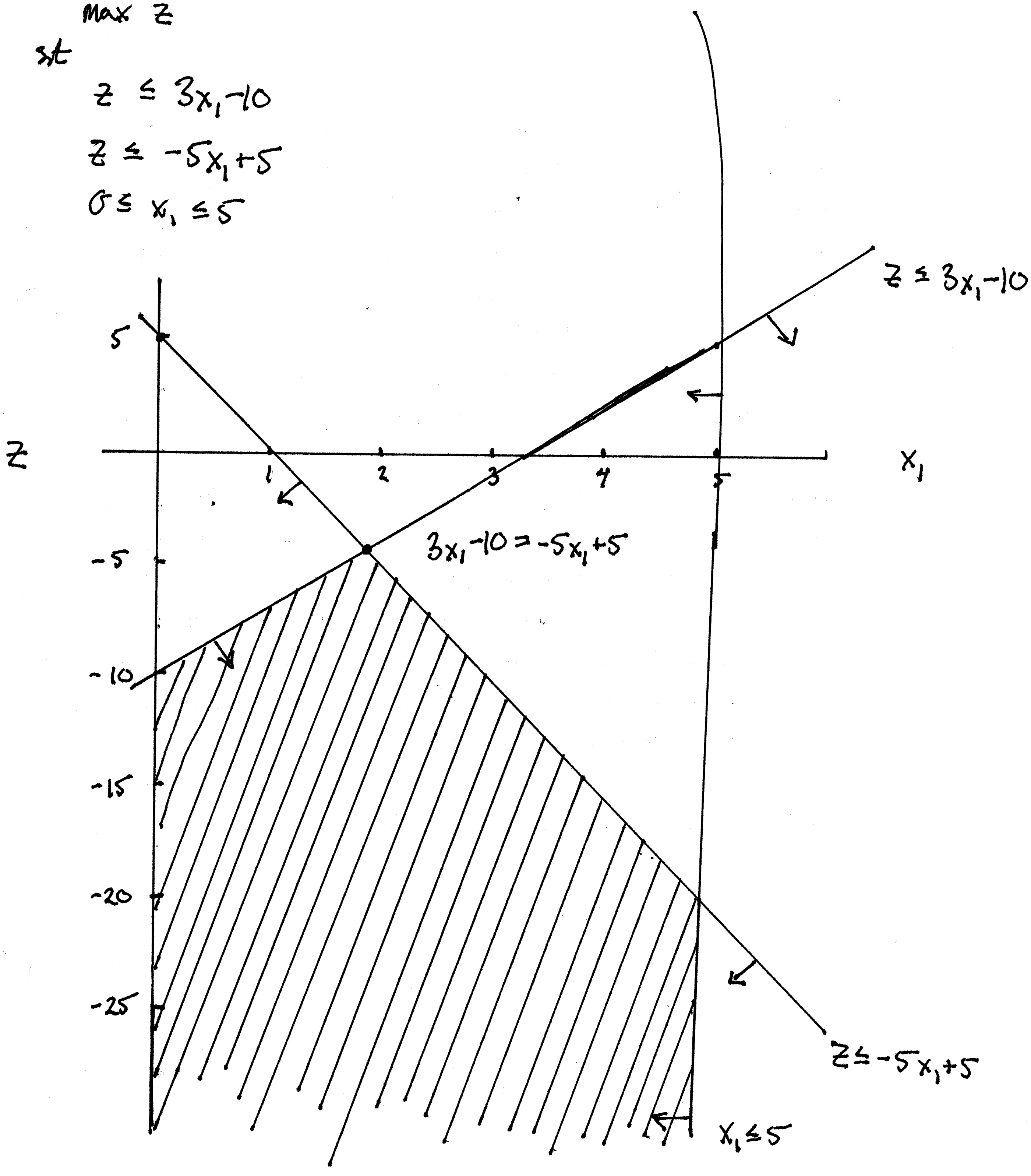
s.t. $\quad 0 \le x_1 \le 5$

Reformulate

$$\max \; z$$

st

$$z \le 3x_1 - 10$$
$$z \le -5x_1 + 5$$
$$0 \le x_1 \le 5$$



Optimum occurs at $x_1 = \frac{15}{8}$ with value $z = \frac{-35}{8}$

4) First transform the LP into standard form.

$$\max\ 2x_1 - 4x_2 + 5x_3 - 6x_4$$
s.t.
$$x_1 + 4x_2 - 2x_3 + 8x_4 + s_1 \quad = 2$$
$$-x_1 + 2x_2 + 3x_3 + 4x_4 \quad + s_2 = 1$$
$$x_1, x_2, x_3, x_4, s_1, s_2 \geq 0$$

4a) Since there are 6 variables and 2 constraints there are at most $15 = \binom{6}{2}$ basic solutions.

4b) The basic solutions are:

* $(0, 1/2, 0, 0, 0, 0)$
* $(8, 0, 3, 0, 0, 0)$
* $(0, 0, 0, 1/4, 0, 0)$
  $(-1, 0, 0, 0, 3, 0)$
* $(2, 0, 0, 0, 0, 3)$
* $(0, 0, 1/3, 0, 8/3, 0)$
  $(0, 0, -1, 0, 0, 4)$
* $(0, 0, 0, 0, 2, 1)$

Of these, those marked with * are feasible and hence feasible extreme points.

4c) By inspection of the list $(8, 0, 3, 0, 0, 0)$ is the optimal basic feasible solution.

5) First transform into standard form.

$$\max \quad 2x_1 + x_2 - 3x_3 + 5x_4$$

s.t.

$$x_1 + 7x_2 + 3x_3 + 7x_4 + s_1 \leq 46$$
$$3x_1 - x_2 + x_3 + 2x_4 + s_2 = 8$$
$$2x_1 + 3x_2 - x_3 + x_4 + s_3 = 10$$
$$x_1, x_2, x_3, x_4, s_1, s_2, s_3 \geq 0$$

The initial simplex tableau with basis s_1, s_2, s_3 is:

$$\begin{array}{rl}
 & \begin{bmatrix} -2 & -1 & 3 & -5 & 0 & 0 & 0 & 0 \\ 1 & 7 & 3 & 7 & 1 & 0 & 0 & 46 \\ 3 & -1 & 1 & 2 & 0 & 1 & 0 & 8 \\ 2 & 3 & -1 & 1 & 0 & 0 & 1 & 10 \end{bmatrix} \\
\end{array}$$

$46/7 > 6$

$8/2 = 4 \rightarrow$ (pivot row 3)

$10/1 = 10$

↑ (pivot column 4)

$$\begin{bmatrix} 11/2 & -7/2 & 11/2 & 0 & 0 & 5/2 & 0 & 20 \\ -19/2 & 21/2 & -1/2 & 0 & 1 & -7/2 & 0 & 18 \\ 3/2 & -1/2 & 1/2 & 1 & 0 & 1/2 & 0 & 4 \\ 1/2 & 7/2 & -3/2 & 0 & 0 & -1/2 & 1 & 6 \end{bmatrix}$$

$36/21 = 12/7$

$12/7 \rightarrow$ (pivot row 4)

↑ (pivot column 2)

$$\begin{bmatrix} 6 & 0 & 8/2 & 0 & 0 & 4/2 & 1 & 26 \\ -11 & 0 & 4 & 0 & 1 & -2 & -3 & 0 \\ 11/7 & 0 & 2/7 & 1 & 0 & 3/7 & 1/7 & 34/7 \\ 1/7 & 1 & -3/7 & 0 & 0 & -1/7 & 2/7 & 12/7 \end{bmatrix}$$

Thus the optimal value is 26 achieved by $(0, 12/7, 0, 34/7, 0, 0, 0)$.

6) First turn into a maximization LP in standard form.

$$-\max \; -x_1 + 3x_2 + 2x_3$$

$$\text{s.t.} \quad \begin{aligned} 3x_1 - x_2 + 2x_3 + s_1 &= 7 \\ -2x_1 + 4x_2 \qquad + s_2 &= 12 \\ -4x_1 + 3x_2 + 8x_3 + s_3 &= 10 \\ x_1, x_2, x_3, s_1, s_2, s_3 &\geq 0 \end{aligned}$$

The initial simplex tableau with basis s_1, s_2, s_3 is:

$$\begin{array}{c c} & \\ & \\ 12/4 = 3 & \rightarrow \\ 10/3 > 3 & \\ \end{array} \begin{bmatrix} 1 & -3 & -2 & 0 & 0 & 0 & 0 \\ 3 & -1 & 2 & 1 & 0 & 0 & 7 \\ -2 & 4 & 0 & 0 & 1 & 0 & 12 \\ -4 & 3 & 8 & 0 & 0 & 1 & 10 \end{bmatrix}$$

$$\begin{array}{c c} 10/2 = 5 & \\ & \\ & \\ 1/8 & \rightarrow \\ \end{array} \begin{bmatrix} -\frac{1}{2} & 0 & -2 & 0 & \frac{3}{4} & 0 & 9 \\ \frac{5}{2} & 0 & 2 & 1 & \frac{1}{4} & 0 & 10 \\ -\frac{1}{2} & 1 & 0 & 0 & \frac{1}{4} & 0 & 3 \\ \frac{5}{2} & 0 & 8 & 0 & -\frac{3}{4} & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{8} & 0 & 0 & 0 & \frac{9}{16} & 2 & 11 \\ \frac{15}{8} & 0 & 0 & 1 & \frac{7}{16} & -\frac{1}{4} & \frac{39}{4} \\ -\frac{1}{2} & 1 & 0 & 0 & \frac{1}{4} & 0 & 3 \\ \frac{5}{16} & 0 & 1 & 0 & -\frac{3}{32} & \frac{1}{8} & \frac{1}{8} \end{bmatrix}$$

Thus the optimal solution is $(0, 3, 1/8, 39/4, 0, 0)$ with value -11.