Here is a list of questions I have asked on previous exams pertaining to topics covered so far in this course as well as some new questions I have created. Note that just because a topic from those sections is not represented here, does *not* mean it will not be tested on. This list is simply meant to be representative.

- Solve the following recurrence relation

$$\begin{aligned} a_n &= 5a_{n-1} - 6a_{n-2} + (7-2n)2^{n-1} \\ a_0 &= 2 \\ a_1 &= 1. \end{aligned}$$

- Let $a_n = 4a_{n-1} + 8a_{n-2} + 7a_{n-3}$, with $a_0 = 0$, $a_1 = 1$, and $a_2 = -1$ Find the generating function for a_n.

- Solve $a_n = 3a_{n-1} - 2a_{n-2} - n2^n$ when $a_0 = -2$ and $a_1 = -1$.

- Find generating functions for a_n and b_n, where $a_0 = 1$, $b_0 = 3$, and

$$\begin{aligned} a_n &= a_{n-1} - 3b_{n-1} \\ b_n &= -a_{n-1} + 2b_{n-1}. \end{aligned}$$

- Find a_n for $n \geq 2$, when $a_2 = 4, a_3 = 5$ and

$$a_n = a_{n-1} + 6a_{n-2},$$

  for $n \geq 4$.

- Evaluate the sum $\sum_{i=0}^{54} \sum_{j=0}^{54-i} \sum_{k=0}^{54-j-k} \frac{54!}{i!j!k!(54-i-j-k)!} \frac{2^i \sqrt{3}^j}{(-3)^{54-i-j-k}}$

- Prove that for all $n \geq 0$, $\sum_{k=n^2+1}^{(n+1)^2} k = n^3 + (n+1)^3$.

- Determine the number of positive integers solutions to $x_1 + x_2 + x_3 + x_4 = 17$, where $x_1 \in \{1, 2, 3\}$, $4 \leq x_2$, $0 \leq x_3$, $3 \leq x_4$.

- Find the number of one-to-one and onto functions from $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$ to $B = \{a, b, c, d, e, f, g, h\}$.

- Show that $2^n < n!$ for all $n \geq 4$. (Note that $n! = 1 \times 2 \times 3 \times \cdots \times n$.)

- Find the sum of all the coefficients in $(w + x + y + z)^5$. Be sure to explain your work.

- Let $F_0 = F_1 = 1$ and define $F_n = F_{n-1} + F_{n-2}$ for all $n \geq 2$. Prove that $\gcd(F_n, F_{n-1}) = 1$ for all $n \geq 1$.

- Let $T(0) = 1$ and let $T(n) = 1 + \sum_{i=0}^{n-1} T(i)$ for $n \geq 1$. Prove using strong induction that $T(n) = 2^n$ for $n \geq 0$. (Recall that $\sum_{i=0}^{n} r^i = \frac{1-r^{n+1}}{1-r}$.)

- How many solutions are there to $x + y + z = 14$ where $x, y, z \in \mathbb{Z}$, $x \geq 2$, $y \geq -4$, and $1 \leq z \leq 3$.
- Using the 26 letters of the English alphabet, how many 11 letter palindromes are there, whose second letter is a vowel (A,E,I,O,U)? Note that a palindrome is a word that reads the same both forwards and backwards, for instance

  BECDFGFDCEB

  is an 11 letter palindrome whose second letter is a vowel.
- Use induction to show that every positive integer greater than 12 can be writen as a sum of 3's and 7's.
- An Institute committee draws from a pool of 10 faculty, 4 of which can be committee chairs, 7 undergraduate students, and 3 graduate students. How many possible committees are there if every committee has one committee chair, two non-chair faculty members, and two student members with at least one of them being an undergraduate student.
- Show that if n is odd, $n^2 - 1$ is divisible by 8.
- How many ways are there to distribute 15 pieces of candy to Alice, Bob, Charlie, David, and Eve if Bob gets at least 3 pieces and Eve is diabetic so can only have 0, 1, or 2 pieces.
- What is the coefficient of $x^{17}z^3w^5$ in $(2x - 3z + 7w)^{25}$.
- How many ways are there to deal out the entire 52 card deck to 4 players? (Note that the players are considered distinct.)
- Find, for all n, the number of n-letter palindromes where every 3^{rd} letter is a vowel, and the rest are consonants. More specifically, we wish there to be precisely 2 consonants between every two vowels.
- Let $a_n = a_{n-1} + a_{n-2}$ with $a_0 = a_1 = 1$. Determine the relative rate of growths of a_n and 2^n.
- Order the following functions in terms of rate of growth.

$$\log(n), n^2 + n, \frac{1}{n}, \frac{1}{n^n}, \log(n!), n^{\frac{3}{2}} \log(n^\pi)$$

- Suppose the running time of some algorithm is at most T_n and that $T_n \leq T_{n-1} + 3n^2$ and $T_0 = 1000$. What is order of T_n.
- Consider the following method of sorting:

  1. Find the smallest element of the list via linear search.
  2. Place the smallest element of the list first.

3. Recurssively sort the remainder of the list.

How does this searching algorithm compare with BubbleSort? Is there any case where this algorithm would be asymptotically better than BubbleSort? What about asymptotically worse?