Here is a list of terms you will be assumed to be familiar with, not that this list may not be complete.

- subset
- graph
- isomorphic
- connected
- degree (sequence)
- planar
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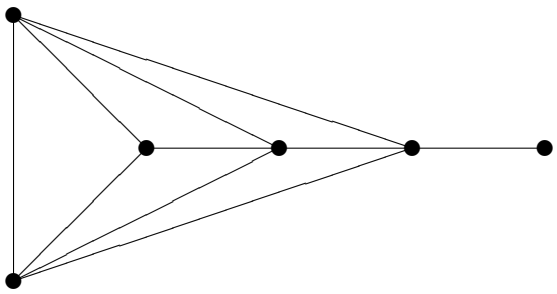
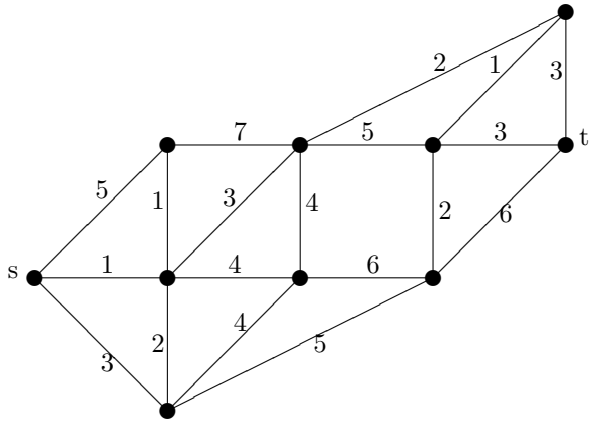
- Find the chromatic number of the following graph.



- Draw a loop-free undirected simple graph on 10 vertices with 20 edges, such that the graph is planar and has an Eulerian circuit. State why the graph drawn satisfies all these properties.
- Alice and Bob are examining a graph with 12 vertices and 31 edges. Alice insists that the graph is planar, but Bob disagrees. Can we tell who is correct, and if so who? Explain your answer in full detail.
- There are four graphs, each of which has exactly one of the following properties; one contains a cycle containing every vertex, one contains an Eulerian circuit, one is planar, and one is has multiple edges. These graphs have the following degree sequences;
  - $7, 7, 7, 7, 7, 7, 6, 6$
  - $2, 2, 4, 4, 6, 6, 6, 6$
  - $4, 4, 4, 4, 4, 4, 5, 9$
  - $3, 3, 3, 3, 3, 3, 4, 4$.

  Identify which property is associated with the graph for each of the degree sequence and why.

- Find the shortest path from s to t in the following graph. No credit will be given if you do not clearly state a proven methodology and show all steps. Note that $a < b < c$.



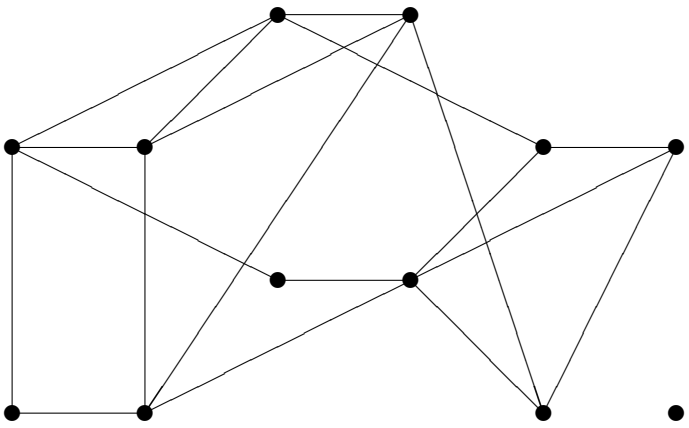
- Determine the number of positive integers solutions to $x_1 + x_2 + x_3 + x_4 = 17$, where $x_1 <= 5$, $4 <= x_2 <= 9$, $x_3 <= 2$, $3 <= x_4 <= 7$.

- Find the number of one-to-one and onto functions from $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$ to $B = \{a, b, c, d, e, f, g, h\}$.

- Find the sum of all the coefficients in $(w + x + y + z)^5$. Be sure to explain your work.

- Prove that if $G = (V_G, E_G)$ and $H = (V_H, E_H)$ are isomorphic graphs, then they have the same number of triangles. In otherwords, they have the same number of induced K_3 subgraphs.

- Recall that a coloring of a graph is an assignment of colors to each vertex, so that no edge goes between vertices of the same color. Color this graph with the minimum number of colors possible.



- How many ways are there to distribute 15 pieces of candy to Alice, Bob, Charlie, David, and Eve if Bob gets at least 3 pieces and Eve is diabetic so can only have 0, 1, or 2 pieces.
- What is the coefficient of $x^{17}z^3w^5$ in $(2x - 3z + 7w)^{25}$.
- How many ways are there to deal out the entire 52 card deck to 4 players? (Note that the players are considered distinct.)
- Let S be a set of five positive integers, the maximum of which is at most 8, show that the sums of the elements in all the nonempty subsets of S cannot all be distinct.
- A bag contains 6 marbles, each marble is equally likely to be red, blue or green. A single marble is pulled out of the bag, given that it is green, what is the probability that the bag contained two green marbles at least 2 blue marbles.
- Consider a graph on $\{1, 2, \cdots, n\}$ where each edge is present independently with probability p. What is the probability that there is a path of length at most two from 1 to n.
- A chemical company needs to store many different chemicals, some of them react violently with each other and so need to be stored in differing rooms. Below is a matrix of which chemicals interact with other chemicals, how

many rooms does the chemical company need?

$$\begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \end{bmatrix}$$