

CSE 167 - Intro to Computer Graphics

ANSWER KEY: Midterm Examination

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Write your name or initials on every page before beginning the exam.

You have 75 minutes. There are eight problems. You may not use “cheat” sheets, calculators, notes, textbooks, or other materials during this exam. You must show your work in order to get credit. Good luck!

Name:

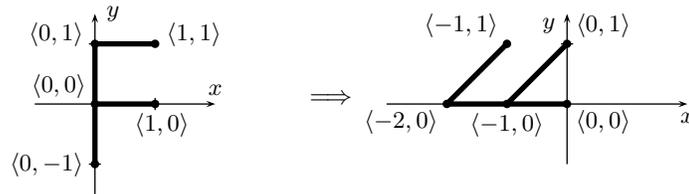
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1. An affine transformation $A(x) : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ maps the “F”-shape on the left to the “F” on the right.



- a. Give a 3×3 matrix that represents A over homogeneous coordinates.

ANSWER: $A = \begin{pmatrix} 1 & -1 & -1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

- b. Give a 3×3 matrix that represents A^{-1} over homogeneous coordinates.

ANSWER: $A^{-1} = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$.

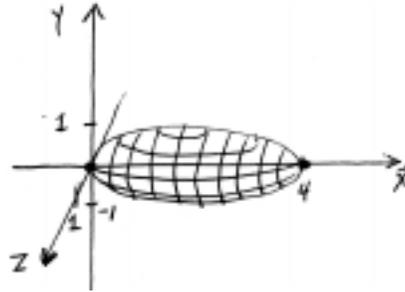
2. Consider the following sequence of OpenGL commands:

```
glMatrixMode(GL_MODELVIEW);  
glLoadIdentity();  
glRotatef( 90.0, 0, 1, 0 );  
glRotatef( 90.0, 0, 0, 1 );  
glRotatef( -90.0, 0, 1, 0 ); // Watch out: negative sign!  
glTranslatef( 3, 4, 5 );
```

Show explicitly the contents of the 4×4 ModelView matrix after these commands have been executed.

ANSWER: $\begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 0 & -1 & -5 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$. *You must be careful here to note that the `glTranslatef` comes after the `glRotatef`'s.*

3. An ellipsoid has 2 radii equal to 1 and one radius equal to 2. It can be viewed as a unit sphere which has been stretched to twice its usual width and stretches from $\mathbf{0} = \langle 0, 0, 0 \rangle$ to $\langle 4, 0, 0 \rangle$. The ellipsoid has circular crosssections when intersected with planes parallel to the yz -plane.



The GLUT command `glutSolidSphere(r, 10, 10);` can be used to generate a sphere of radius r . Using this command, plus other OpenGL commands, such as `glMatrixMode`, `glLoadIdentity`, `glRotatef`, `glScalef`, `glTranslatef`, write a code fragment that will generate the ellipsoid.

ANSWER: `glMatrixMode(GL_MODELVIEW);`
`glLoadIdentity();`
`glTranslatef(2.0, 0.0, 0.0);`
`glScalef(2.0, 1.0, 1.0);`
`glutSolidSphere(1.0, 10, 10);`

4. What are the different kinds of light from point light sources in the Phong lighting model? Give a short description of each one (one sentence, and possibly a simple picture).

ANSWER:

Specular light Light that reflects primarily in the direction of perfect reflection (see figure III.3, on page 69).

Diffuse light Light that reflects with equal brightness in all directions (see figure III.2, on page 69).

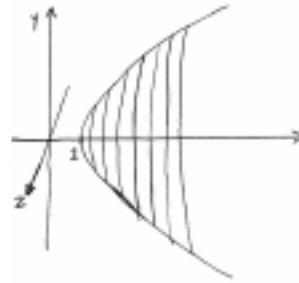
Ambient light This is not really from a point light source, but no points were taken off if it was mentioned, since the OpenGL implementation of Phong lighting does include ambient light from point light sources.

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5. The hyperboloid H is the set of points defined by

$$H = \{(x, y, z) : x^2 - y^2 - z^2 = 1, x > 0\}.$$



Suppose $\mathbf{p} = \langle p_0, p_1, p_2 \rangle$ is a point on H . Give a formula for a unit vector normal to the surface H at the point \mathbf{p} .

ANSWER: Let $f(x, y, z) = x^2 - y^2 - z^2$. Then $\nabla f = \langle 2x, -2y, -2z \rangle$. A unit vector normal to the surface $\mathbf{p} = \langle p_0, p_1, p_2 \rangle$ is given by

$$\frac{\nabla f(\mathbf{p})}{\|\nabla f(\mathbf{p})\|} = \frac{\langle p_0, -p_1, -p_2 \rangle}{\sqrt{p_0^2 + p_1^2 + p_2^2}}.$$

6. Describe the difference between Gouraud shading and Phong shading. Which one is commonly used by OpenGL? What are the relative advantages of Gouraud shading over Phong shading? What are the relative advantages of Phong shading over Gouraud shading?

ANSWER: Gouraud shading computes the colors at each vertex of a triangle (or quad or polygon) with the Phong lighting model and interpolates their colors into the interior of the shape. Phong shading interpolates normal vectors from the vertices into the interior of the shape, and recomputes illumination levels (i.e., color values) of each pixel with the Phong lighting model.

Gouraud shading is commonly used by OpenGL.

Advantages of Gouraud shading are speed, simplicity and ease of implementation.

Advantages of the Phong shading: Much better at catching specular highlights and spotlights. Better results with rendering with large polygons and when animating a scene.

The disadvantage of Phong lighting is that information about the lights, and the material properties, etc., must be remembered until late in the graphics rendering pipeline.

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7. Let $\mathbf{x} = \langle 2, -2 \rangle$ and $\mathbf{y} = \langle -1, 1 \rangle$.

(a) What is the point $\mathbf{u}_1 = \text{Lerp}(\mathbf{x}, \mathbf{y}, 1)$?

ANSWER: $\langle -1, 1 \rangle$.

(b) What is the point $\mathbf{u}_2 = \text{Lerp}(\mathbf{x}, \mathbf{y}, \frac{1}{2})$?

ANSWER: $\langle \frac{1}{2}, -\frac{1}{2} \rangle$.

(c) What is the point $\mathbf{u}_3 = \text{Lerp}(\mathbf{x}, \mathbf{y}, -2)$?

ANSWER: $\langle 8, -8 \rangle$.

(d) Let $\mathbf{u}_4 = \langle 1, -1 \rangle$. Find the value of α such that $\text{Lerp}(\mathbf{x}, \mathbf{y}, \alpha) = \mathbf{u}_4$.

ANSWER: $\alpha = \frac{1}{3}$.

8. Let $\mathbf{x} = \langle -1, 0, 5 \rangle$ and $\mathbf{y} = \langle 0, 5, -1 \rangle$ and $\mathbf{z} = \langle 4, 1, -2 \rangle$ be the vertices of triangle. Consider barycentric coordinates relative to this triangle.

(a) What point \mathbf{v}_1 has barycentric coordinates $\langle \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \rangle$?

ANSWER: $\langle 1, 2, \frac{2}{3} \rangle$.

(b) What point \mathbf{v}_2 has barycentric coordinates $\langle 1, 0, 0 \rangle$?

ANSWER: $\langle -1, 0, 5 \rangle$.

(c) What point \mathbf{v}_3 has barycentric coordinates $\langle \frac{1}{4}, \frac{1}{4}, \frac{1}{2} \rangle$?

ANSWER: $\langle \frac{7}{4}, \frac{3}{2}, 0 \rangle$.

(d) What point \mathbf{v}_4 has barycentric coordinates $\langle \frac{1}{3}, \frac{2}{3}, 0 \rangle$?

ANSWER: $\langle \frac{-1}{3}, \frac{10}{3}, 1 \rangle$.

(e) What are the barycentric coordinates of the point \mathbf{y} ?

ANSWER: $\langle 0, 1, 0 \rangle$.