1. Suppose the earth is to be drawn at the origin and it is to rotate on its axis at a rate of $\omega$ degrees per unit time. The axis of rotation is to be the $y$-axis. Give the OpenGL commands that will set the model view matrix correctly to render the earth at time $t$. *Hint: Your answer will need to include a rotation through an angle of $\omega t$ degrees.*

2. Suppose the sun is positioned at the origin and the earth is revolving around the sun at a rate of $\omega$ degrees per unit time and at a distance of 5 units. Also suppose same side of the earth is always facing the sun. The earth always stays in the $xz$-plane. Give the OpenGL commands that will set the model view matrix correctly to render the earth at time $t$. *Hint: Your answer will need to include a rotation through an angle of $\omega t$ degrees.*

3. Let $\mathbf{u} = (0, 1, 0)$. Consider the rotation $R_{90^\circ, \mathbf{u}}$. Give a $4 \times 4$ homogeneous matrix that represents $R_{90^\circ, \mathbf{u}}$.

4. What will the model view matrix equal after the following OpenGL commands?

   ```
   glMatrixMode(GL_MODELVIEW);
   glLoadIdentity();
   glRotatef(90.0, 0, 1, 0);
   glRotatef(90.0, 0, 0, 1);
   ```

   What will it equal after the commands:

   ```
   glMatrixMode(GL_MODELVIEW);
   glLoadIdentity();
   glRotatef(90.0, 0, 0, 1);
   glRotatef(90.0, 0, 1, 0);
   ```

5. Let $M$ be the matrix

   $\begin{pmatrix}
   0 & 0 & 1 \\
   1 & 0 & 0 \\
   0 & 1 & 0
   \end{pmatrix}$. 
Let $A(x)$ be the transformation of $\mathbb{R}^3$ defined by $A(x) = Mx$. Give a sequence of OpenGL commands that will cause the same effect as the transformation $A$ (without explicit loading the entries of the matrix!).

*Hint: it can be done fairly easily using two rotations.*

6. Let $A$ and $M$ be as in the previous problem. Redo question 4., but use only a single rotation. This is mathematically equivalent to expressing $A$ in the form $A = R_{\theta,u}$ where $u$ is a unit vector.

*Hint: You can find $u$ without any hard calculations by using symmetry. The value of $\theta$ can be found from the fact that $A \circ A \circ A$ is equal to the identity transformation.*
Selected answers

1. The following commands can be used:

```c
glMatrixMode(GL_MODELVIEW);
glLoadIdentity();
glRotatef( \omega t, 0, 1, 0 );
drawEarth();
```

2. The following commands can be used:

```c
glMatrixMode(GL_MODELVIEW);
glLoadIdentity();
glRotatef( \omega t, 0, 1, 0 );
glTranslatef( 5, 0, 0 );
drawEarth();
```

4. The matrices are different! They are

\[
\begin{pmatrix}
0 & 0 & 1 & 0 \\
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 \\
\end{pmatrix}
\quad \text{and} \quad
\begin{pmatrix}
0 & -1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
-1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 \\
\end{pmatrix}
\]

You should try visualizing the effects of these transformations and understand why they are different.

5. Further hint: One way to achieve this transformation is with a 90 degree rotation around the \( z \)-axis, followed by a 90 degree rotation around the \( y \)-axis.