Write your name or initials on every page before beginning the exam.

You have 75 minutes. There are six problems. You may not use calculators, notes, textbooks, or other materials during this exam. You must show your work in order to get credit. Good luck!
1. As shown in the figure below, an “F” placed in standard position at the origin is transformed by an transformation \( A \). \( A \) is an affine transformation of \( \mathbb{R}^2 \). The effect of \( A \) on the “F” can be informally described as follows: The “F” is translated a distance \( \ell \) from the origin along a line that makes angle \( \theta \) with the \( x \)-axis. It is then tilted through an angle of \( \phi \). (You may not want to implement the transformation in this way your OpenGL program, however.) Let \texttt{drawF()} be a routine that draws the “F” in standard position. Use \texttt{drawF} and (pseudo)-OpenGL commands to draw the “F” as transformed by \( A \).

One possible answer:

```cpp
glMatrixMode(GL_MODELVIEW);
glLoadIdentity();
pglRotate(\theta);
pglTranslate(\ell,0);
pglRotate(\phi - \theta);
drawF();
```
2. Write OpenGL commands that generate a solid “F” formed from three ellipsoids as in the figure. The three ellipsoids have circular cross sections: their central circular cross-sections are circles of diameter 1 (radius $\frac{1}{2}$). The lengths of the three ellipsoids are all different: the endpoints of the ellipsoids are as labelled.

Write a fragment of an OpenGL program that generates the solid “F”. Use a command `drawUnitSphere()` that draws a sphere of radius one centered at the origin and use OpenGL commands such as `LoadIdentity()`, `glTranslatef()`, `glRotatef()`, `glScalef()`, `glLoadIdentity()`, `glMultMatrix()`, `glPushMatrix()`, and `glPopMatrix()`.

Possible answer:

```glsl
glMatrixMode(GL_MODELVIEW);
gLoadIdentity();
gPushMatrix();
gTranslatef(0,2,0);
gScalef(0.5,2,0.5);
drawUnitSphere();
gPopMatrix();
gPushMatrix();
gTranslatef(1,4,0);
gScalef(1,0.5,0.5);
drawUnitSphere();
gPopMatrix();
gPushMatrix();
gTranslatef(1.25,2,0);
gScalef(0.75,0.5,0.5);
drawUnitSphere();
gPopMatrix();
```

3. In the figure to the right, label the vectors $\ell$, $v$ and $n$. Then draw and label clearly the halfway vector and the reflection direction vector. Give the formulas for calculating these two vectors from $n$, $\ell$, and $v$.

Answer: $h = \frac{\ell + v}{||\ell + v||}$; $r = 2(\ell \cdot n)n - \ell$.

4. Give the complete Phong lighting equation for the intensity of visible light, for a single wavelength (color) and a single light source. Use the halfway vector formulation for the specular light calculation.

Answer: $I = \rho_a I_a + \rho_d I_d^m (\ell \cdot n) + \rho_s I_s^m (h \cdot n)^f + I_e$. 
5. Discuss the depth buffer algorithm. Describe the algorithm. What is its purpose? How is it implemented? What are its advantages over competing algorithms? What are its disadvantages relative to competing algorithms? (Be specific about what competing algorithms you are discussing.)

Answer:
Purpose: To not display hidden surfaces.
Implemented: with a depth value stored per pixel. Comparison of depth values determines which surface to be displayed.
Advantages over Painter’s Algorithm: (1) Does not require sorting polygons. Can render polygons in any order. (2) Can handle interpenetrating surfaces and the cases where surfaces cannot be sorted by depth order (e.g., three cyclically overlapping surfaces).
Advantages over geometric algorithm: Much simpler and easier to code.
Advantages over both: Renders each polygon independently and fits well into the scheme of pipelined algorithms implemented in graphics hardware.
Disadvantages: (1) Memory usage, although this is less of problem now that memory is so cheap. (2) z-fighting caused by floating point accuracy. (3) Near and far clipping is needed.

6. Let \( \mathbf{u} \) be the unit vector \( (\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}) \). Consider the transformation \( \mathbf{v} \mapsto \text{Proj}_u(\mathbf{v}) \) that sends a vector \( \mathbf{v} \) to its projection onto \( \mathbf{u} \). This is a linear transformation of \( \mathbb{R}^3 \). Give a 3 \( \times \) 3 matrix that represents this linear transformation.

Answer:
\[
\begin{pmatrix}
\frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\
\frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\
\frac{1}{3} & \frac{1}{3} & \frac{1}{3}
\end{pmatrix}
\]