1. Let the flattened ellipsoid $E$ have radii 2, 1, 2, and be defined by the equation $x^2 + 4y^2 + z^2 = 4$. Where does the ellipsoid intersect the three axes?
   a. Draw a picture by hand, illustrating $E$ as best you can.
   b. Suppose $(x, y, z)$ is a point on the ellipse. Give a formula for the outward normal at the ellipse at this point.
   b. Give a parametric equation for $E$; that is, a function $f$ of two parameters (for instance, $\theta$ and $\phi$) so that $f(\theta, \phi)$ gives the points on the ellipsoid as its two parameters vary. What ranges do the parameters need to vary over? *Hint: do this similarly to the parametric description of a sphere in spherical coordinates.*
   c. Use the parametric equation for $E$ to give a formula for the outward unit normal at a point on $E$. Your formula for the normal should be in terms of $\theta$ and $\phi$.

2. A cone $C$ has tip at $(0, 1, 0)$, and its base is the disk $x^2 + z^2 \leq 1$ in the $xz$-plane. Suppose $(x, y, z)$ is a point on the side of the ellipse (not on the base). Give a formula for the outward unit normal to the cone at this point.

3. Let $x = (-2, 2)$ and $y = (2, 0)$. Let $\alpha$ control linear interpolation and extrapolation from $x$ to $y$. What five points are obtained when $\alpha = -2$, $\alpha = 0$, $\alpha = \frac{1}{2}$, $\alpha = 1$, and $\alpha = 2$? What value of $\alpha$ gives the point $(1.6, 0.2)$? Graph by hand all these values, labeling things clearly.

4. Continuing problem 3. If you apply the formula for inverting linear interpolation to the origin $(0, 0)$, what value of $\alpha$ do you get? Use this to compute the point on the line containing $x$ and $y$ that is closest to the origin.

5. Let $x = (-1, -2)$, and $y = (1, 1)$, and $z = (1, -1)$. What points are obtained by the following sets of barycentric coordinates?
   a. $\alpha = 0$, $\beta = 0$, and $\gamma = 1$.
   b. $\alpha = \frac{2}{3}$, $\beta = \frac{1}{3}$, and $\gamma = 0$.
   c. $\alpha = \frac{1}{3}$, $\beta = \frac{1}{3}$, and $\gamma = \frac{1}{3}$.
   d. $\alpha = \frac{4}{5}$, $\beta = \frac{1}{10}$, and $\gamma = \frac{1}{10}$.
   e. $\alpha = \frac{1}{3}$, $\beta = \frac{2}{3}$, and $\gamma = -1$. 
6. For the same triangle as in the previous problem, what are the barycentric coordinates of the following points?
   a. \( u = (1, 1) \).
   b. \( u = (\frac{1}{3}, 0) \).
   c. \( u = (\frac{1}{2}, -\frac{1}{2}) \).

7. For the same triangle again: Draw a graph showing where the points lie that have barycentric coordinates with \( \alpha > 0, \beta > 0, \) and \( \gamma < 0 \).

8. Let \( x = (0, 0, 0) \), \( y = (5, 0, 1) \), \( z = (4, 1, 1) \), and \( w = (-1, 2, 0) \) be the four vertices of a quadrangle in counterclock-wise order. For each pair of values \( \alpha \) and \( \beta \), what point is obtained by bilinear interpolation in this quadrangle? (Or, if no such point exists, explain why not.)
   a. \( \alpha = 0 \) and \( \beta = 1 \).
   b. \( \alpha = 1 \) and \( \beta = 1 \).
   c. \( \alpha = \frac{1}{3} \) and \( \beta = \frac{2}{3} \).
   d. \( \alpha = \frac{1}{3} \) and \( \beta = \frac{1}{3} \).