Math 155A — Computer Graphics — Winter 2019
Homework #1 — Due Tuesday, January 29, 9:00pm
Hand in via Gradescope — Use separate pages for each problem 1.-5.

References to figures are mostly in draft A.2.c of the second edition.
(At http://www.math.ucsd.edu/ sbuss/CourseWeb/Math155A_2019Winter/SecondEdDraft.pdf.)

1. (a) Define functions

- $f_1 : \langle x_1, x_2 \rangle \mapsto \langle x_2, -x_1 \rangle$, and
- $f_2 : \langle x_1, x_2 \rangle \mapsto \langle 2x_1, -\frac{1}{2}x_2 \rangle$, and
- $f_3 : \langle x_1, x_2 \rangle \mapsto \langle x_1 - \frac{1}{2}x_2, x_2 \rangle$, and
- $f_4 : \langle x_1, x_2 \rangle \mapsto \langle x_2, x_1 - \frac{1}{2}x_2 \rangle$.

Verify (but do not show your work) that each f_i is linear. Draw figures showing how these four functions transform the "F"-shape. Your four figures should be similar to the drawing on the right in Figure II.3 on page 22 in the first edition, or in the drawings Figure II.3 on page 36 (and Figures II.4 and II.5 on page 40) of the second edition draft A.2.c.

(b) Consider the functions $f_1, \ldots, f_4$ from problem 1. Which of the four f_i's are affine? Which of the four f_i's are rigid? Which of the f_i's are orientation-preserving?

2. Give the 2×2 matrices representing the linear transformations in $\mathbb{R}^2$ in parts (b), (c) and (e) of Figure II.3 on page 36 (second edition draft A.2.c). They are a uniform scaling $S_{\frac{1}{2}}$, a nonuniform scaling $S_{\langle \frac{3}{2}, \frac{1}{2} \rangle}$ and a shearing transformation.

3. (a) Let f be the affine transformation of $\mathbb{R}^2$ shown in Figure II.5 on page 40 of draft A.2.c. Express this function as an affine function by writing it explicitly in the form $f(\mathbf{x}) = M\mathbf{x} + \mathbf{u}$ where M is a 2×2 matrix. Then give its 3×3 matrix represention.

(b) Now express its inverse f^{-1} in the form $N\mathbf{x} + \mathbf{v}$ where N is a 2×2 matrix. Second, give the 3×3 matrix representing the inverse f^{-1} of f.

4. Let $T_{\mathbf{u}}$ and $R_{\theta,\mathbf{u}}$ be the translation and rotation operators in $\mathbb{R}^3$. Give the 4×4 matrix representations of $T_{\mathbf{i}-\mathbf{k}} \circ R_{\pi/2,\mathbf{j}}$ and of $R_{\pi/2,\mathbf{j}} \circ T_{\mathbf{i}-\mathbf{k}}$. (You can do this either by multiplying matrices or by visualizing. If you do it with matrices, you are highly recommended to try visualizing afterwards.)

5. A $2 \times 2 \times 2$ cube has the eight vertices $\langle \pm 1, \pm 1, \pm 1 \rangle$. Show how to render the six faces of the cube with two triangle fans, by explicitly listing the vertices used for the triangle fans. Make sure that the usual CCW front faces are facing outwards. (There is more than one way to do this, but please use $\langle 1, 1, 1 \rangle$ as the first (central) vertex for one of the triangle fans.)