1. (a) Define functions
   \[ f_1: \langle x_1, x_2 \rangle \mapsto \langle x_2, -x_1 \rangle, \quad f_2: \langle x_1, x_2 \rangle \mapsto \langle 2x_1, -\frac{1}{2}x_2 \rangle, \]
   \[ f_3: \langle x_1, x_2 \rangle \mapsto \langle x_1 - \frac{1}{2}x_2, x_2 \rangle, \quad f_4: \langle x_1, x_2 \rangle \mapsto \langle x_2, x_1 - \frac{1}{2}x_2 \rangle. \]
   Verify (but do not show your work) that each \( f_i \) is linear. Draw figures showing how these four functions transform the "F"-shape. Your four figures should be similar to the drawing on the right in Figure II.3 on page 22 in the first edition, or in the drawings Figure II.3 on page 36 (and Figures II.4 and II.5 on page 40) of the second edition draft A.2.c.

(b) Consider the functions \( f_1, \ldots, f_4 \) from problem 1. Which of the four \( f_i \)'s are affine? Which of the four \( f_i \)'s are rigid? Which of the \( f_i \)'s are orientation-preserving?

2. Give the \( 2 \times 2 \) matrices representing the linear transformations in \( \mathbb{R}^2 \) in parts (b), (c) and (e) of Figure II.3 on page 36 (second edition draft A.2.c). They are a uniform scaling \( S_{\frac{1}{2}} \), a nonuniform scaling \( S_{\langle \frac{3}{2}, \frac{1}{2} \rangle} \) and a shearing transformation.

3. (a) Let \( f \) be the affine transformation of \( \mathbb{R}^2 \) shown in Figure II.5 on page 40 of draft A.2.c. Express this function as an affine function by writing it explicitly in the form \( f(x) = Mx + u \) where \( M \) is a \( 2 \times 2 \) matrix. Then give its \( 3 \times 3 \) matrix representation.

(b) Now express its inverse \( f^{-1} \) in the form \( Nx + v \) where \( N \) is a \( 2 \times 2 \) matrix. Second, give the \( 3 \times 3 \) matrix representing the inverse \( f^{-1} \) of \( f \).

4. Let \( T_u \) and \( R_{\theta,u} \) be the translation and rotation operators in \( \mathbb{R}^3 \). Give the \( 4 \times 4 \) matrix representations of \( T_{1-k} \circ R_{\pi/2,j} \) and of \( R_{\pi/2,j} \circ T_{1-k} \). (You can do this either by multiplying matrices or by visualizing. If you do it with matrices, you are highly recommended to try visualizing afterwards.)

5. A \( 2 \times 2 \times 2 \) cube has the eight vertices \( \langle \pm 1, \pm 1, \pm 1 \rangle \). Show how to render the six faces of the cube with two triangle fans, by explicitly listing the vertices used for the triangle fans. Make sure that the usual CCW front faces are facing outwards. (There is more than one way to do this, but please use \( \langle 1,1,1 \rangle \) as the first (central) vertex for one of the triangle fans.)