Math 155A — Computer Graphics — Winter 2019
Homework #3 — Due Wednesday, February 20, 9:00pm
Hand in via Gradescope — Use separate pages for each problem.

1. Let $h(r) = 1 - r^2$ define a surface of revolution $\mathbf{f}(r,\theta)$ by revolving the graph of h around the y-axis. Give the formula for a point $\mathbf{f}(\theta, r)$ on the surface of rotation. Also give a formula for a normal vector (not necessarily a unit vector) at that point on the surface of rotation. Your normal vector should be pointing generally upward.

2. Let S be the paraboloid surface

$$S \;=\; \{\langle x,y,z\rangle : y = 1 - x^2 - z^2\}. \tag{1}$$

Use the gradient method to give a formula $\mathbf{n} = \mathbf{n}(x,y,z)$ for a vector normal to S at a point $\langle x,y,z\rangle$ on S. The vector $\mathbf{n}$ should be pointing generally upward.

Does your answer agree with the answer to the previous problem? If so, how? If not, why not?

3. A smooth surface S in $\mathbb{R}^3$ is transformed by the linear transformation $f : \mathbb{R}^3 \to \mathbb{R}^3$ represented by the matrix

$$N \;=\; \begin{pmatrix} 2 & 0 & 0 \\ 0 & 0 & \frac{1}{2} \\ 0 & 4 & 0 \end{pmatrix}$$

to form the surface $f(S)$. Give a 3×3 matrix M such that whenever $\mathbf{n}$ is normal to S at a point $\mathbf{x}$ on S, then the vector $\mathbf{m} = M\mathbf{n}$ is normal to the point $f(\mathbf{x})$ on the surface $f(S)$. [Hint: It is not difficult to invert N.]

4. The upper half of a hyperboloid $\mathcal{H}$ is specified by the equation $y = \sqrt{1+x^2+2z^2}$. Equivalently, $\mathcal{H} = \{\langle x,y,z\rangle : y^2 = 1 + x^2 + 2z^2,\ y{\geq}0\}$.

(a) Draw a rough sketch of the hyperboloid $\mathcal{H}$.

(b) Suppose $\langle x,y,z\rangle$ is a point on $\mathcal{H}$. Give a formula for a vector normal to $\mathcal{H}$ at the point $\langle x,y,z\rangle$, using crossproducts of partial derivatives). Your vector need not be a unit vector, but it should point downward from the surface (so as to be outward facing for a viewer placed at the origin).