1. Let \( h(r) = 1 - r^2 \) define a surface of revolution \( f(r, \theta) \) by revolving the graph of \( h \) around the \( y \)-axis. Give the formula for a point \( f(\theta, r) \) on the surface of rotation. Also give a formula for a normal vector (not necessarily a unit vector) at that point on the surface of rotation. Your normal vector should be pointing generally upward.

2. Let \( S \) be the paraboloid surface

\[
S = \{ \langle x, y, z \rangle : y = 1 - x^2 - z^2 \}.
\]

Use the gradient method to give a formula \( n = n(x, y, z) \) for a vector normal to \( S \) at a point \( \langle x, y, z \rangle \) on \( S \). The vector \( n \) should be pointing generally upward.

Does your answer agree with the answer to the previous problem? If so, how? If not, why not?

3. A smooth surface \( S \) in \( \mathbb{R}^3 \) is transformed by the linear transformation \( f : \mathbb{R}^3 \to \mathbb{R}^3 \) represented by the matrix

\[
N = \begin{pmatrix}
2 & 0 & 0 \\
0 & 0 & \frac{1}{2} \\
0 & 4 & 0
\end{pmatrix}
\]

to form the surface \( f(S) \). Give a \( 3 \times 3 \) matrix \( M \) such that whenever \( n \) is normal to \( S \) at a point \( \mathbf{x} \) on \( S \), then the vector \( \mathbf{m} = M \mathbf{n} \) is normal to the point \( f(\mathbf{x}) \) on the surface \( f(S) \). [Hint: It is not difficult to invert \( N \).]

4. The upper half of a hyperboloid \( \mathcal{H} \) is specified by the equation \( y = \sqrt{1 + x^2 + 2z^2} \).
Equivalently, \( \mathcal{H} = \{ \langle x, y, z \rangle : y^2 = 1 + x^2 + 2z^2, y \geq 0 \} \).

(a) Draw a rough sketch of the hyperboloid \( \mathcal{H} \).

(b) Suppose \( \langle x, y, z \rangle \) is a point on \( \mathcal{H} \). Give a formula for a vector normal to \( \mathcal{H} \) at the point \( \langle x, y, z \rangle \), using crossproducts of partial derivatives). Your vector need not be a unit vector, but it should point downward from the surface (so as to be outward facing for a viewer placed at the origin).