Math 155A — Computer Graphics — Winter 2019
Homework #4 — Due Wednesday, February 27, 9:00pm
Hand in via Gradescope — Use separate pages for each problem.

1. Let $\mathbf{x}_1 = \langle -1, 0\rangle$ and $\mathbf{x}_2 = \langle 2, 1\rangle$. Let α control the linear interpolation (and linear extrapolation) from $\mathbf{x}_1$ to $\mathbf{x}_2$ by $\text{Lerp}(\mathbf{x}_1, \mathbf{x}_2, \alpha)$.

(a) What points are obtained with α equal to -2, -1, 0, $\frac{1}{10}$, $\frac{1}{3}$, $\frac{1}{2}$, 1, $1\frac{1}{2}$ and 2? What value of α gives the point $\langle 1, \frac{2}{3}\rangle$? The point $\langle 8, 3\rangle$? Graph your answers.

(b) What point $\mathbf{u}$ on the line containin $\mathbf{x}_1$ and $\mathbf{x}_2$ is the closest to the origin? Find the value α such that $\mathbf{u} = \text{Lerp}(\mathbf{x}_1, \mathbf{x}_2, \alpha)$.

(c) Suppose the values for $f(\mathbf{x}_1) = -3$ and $f(\mathbf{x}_2) = 3$ have been set, and we wish to set other values for $f(\mathbf{z})$ by linear interpolation/extrapolation. What will this set $f(\langle 1, \frac{2}{3}\rangle)$ equal to?

2. Let $\mathbf{x} = \langle 0, 0\rangle$, $\mathbf{y} = \langle 2, 3\rangle$, and $\mathbf{z} = \langle 3, 1\rangle$ in $\mathbb{R}^2$. Determine the points represented by the following sets of barycentric coordinates.

a. $\alpha = 0$, $\beta = 1$, $\gamma = 0$.
b. $\alpha = \frac{2}{3}$, $\beta = \frac{1}{3}$, $\gamma = 0$.
c. $\alpha = \frac{1}{3}$, $\beta = \frac{1}{3}$, $\gamma = \frac{1}{3}$.
d. $\alpha = \frac{4}{5}$, $\beta = \frac{1}{10}$, $\gamma = \frac{1}{10}$.
e. $\alpha = \frac{4}{3}$, $\beta = \frac{2}{3}$, $\gamma = -1$.

Graph your answers along with the triangle formed by $\mathbf{x}$, $\mathbf{y}$, and $\mathbf{z}$.

3. Let, again, $\mathbf{x} = \langle 0, 0\rangle$, $\mathbf{y} = \langle 2, 3\rangle$, and $\mathbf{z} = \langle 3, 1\rangle$. Determine the barycentric coordinates of the following points:

a. $\mathbf{u}_1 = \langle 2, 3\rangle$.
b. $\mathbf{u}_2 = \langle 1\frac{1}{3}, 2\rangle$.
c. $\mathbf{u}_3 = \langle \frac{3}{2}, \frac{3}{2}\rangle$.
d. $\mathbf{u}_4 = \langle 1, 0\rangle$.