1. Let \( \mathbf{x}_1 = \langle -1, 0 \rangle \) and \( \mathbf{x}_2 = \langle 2, 1 \rangle \). Let \( \alpha \) control the linear interpolation (and linear extrapolation) from \( \mathbf{x}_1 \) to \( \mathbf{x}_2 \) by \( \text{Lerp}(\mathbf{x}_1, \mathbf{x}_2, \alpha) \).

(a) What points are obtained with \( \alpha \) equal to \(-2, -1, 0, \frac{1}{10}, \frac{1}{3}, \frac{1}{2}, 1, 1\frac{1}{2} \) and \( 2 \)? What value of \( \alpha \) gives the point \( \langle 1, \frac{2}{3} \rangle \)? The point \( \langle 8, 3 \rangle \)? Graph your answers.

(b) What point \( \mathbf{u} \) on the line containing \( \mathbf{x}_1 \) and \( \mathbf{x}_2 \) is the closest to the origin? Find the value \( \alpha \) such that \( \mathbf{u} = \text{Lerp}(\mathbf{x}_1, \mathbf{x}_2, \alpha) \).

(c) Suppose the values for \( f(\mathbf{x}_1) = -3 \) and \( f(\mathbf{x}_2) = 3 \) have been set, and we wish to set other values for \( f(\mathbf{z}) \) by linear interpolation/extrapolation. What will this set \( f(\langle 1, \frac{2}{3} \rangle) \) equal to?

2. Let \( \mathbf{x} = \langle 0, 0 \rangle \), \( \mathbf{y} = \langle 2, 3 \rangle \), and \( \mathbf{z} = \langle 3, 1 \rangle \) in \( \mathbb{R}^2 \). Determine the points represented by the following sets of barycentric coordinates.

   a. \( \alpha = 0, \beta = 1, \gamma = 0 \).
   b. \( \alpha = \frac{2}{3}, \beta = \frac{1}{3}, \gamma = 0 \).
   c. \( \alpha = \frac{1}{3}, \beta = \frac{1}{3}, \gamma = \frac{1}{3} \).
   d. \( \alpha = \frac{1}{5}, \beta = \frac{1}{10}, \gamma = \frac{1}{10} \).
   e. \( \alpha = \frac{4}{5}, \beta = \frac{2}{5}, \gamma = -1 \).

Graph your answers along with the triangle formed by \( \mathbf{x}, \mathbf{y}, \) and \( \mathbf{z} \).

3. Let, again, \( \mathbf{x} = \langle 0, 0 \rangle \), \( \mathbf{y} = \langle 2, 3 \rangle \), and \( \mathbf{z} = \langle 3, 1 \rangle \). Determine the barycentric coordinates of the following points:

   a. \( \mathbf{u}_1 = \langle 2, 3 \rangle \).
   b. \( \mathbf{u}_2 = \langle 1\frac{1}{2}, 2 \rangle \).
   c. \( \mathbf{u}_3 = \langle \frac{3}{2}, \frac{3}{2} \rangle \).
   d. \( \mathbf{u}_4 = \langle 1, 0 \rangle \).