

# Homework 5 answers - Math 155A - Winter 2019

#1 Different specular exponents would mean that specular highlights are different sizes for different colors. Could get a rainbow effect.

For example :



#2 (a)  $\langle 4, 0 \rangle$

$$(b) \frac{2}{3}\vec{w} + \frac{1}{3}\vec{z} = \left\langle \frac{5}{3}, \frac{4}{3} \right\rangle$$

$$(c) \text{lerp}(\text{lerp}(\vec{x}, \vec{y}, \frac{1}{2}), \text{lerp}(\vec{w}, \vec{z}, \frac{1}{2}), \frac{1}{3})$$

$$= \text{lerp}(\langle 2, 0 \rangle, \langle \frac{5}{2}, \frac{5}{2} \rangle, \frac{1}{3}) = \left\langle \frac{13}{6}, \frac{5}{6} \right\rangle$$

$$(d) \text{lerp}(\text{lerp}(\vec{x}, \vec{y}, \frac{2}{3}), \text{lerp}(\vec{w}, \vec{z}, \frac{1}{3}), \frac{1}{3})$$

$$= \text{lerp}(\langle \frac{4}{3}, 0 \rangle, \langle \frac{5}{3}, \frac{7}{3} \rangle, \frac{1}{3}) = \left\langle \frac{13}{9}, \frac{7}{9} \right\rangle.$$

$$\underline{\underline{\#3}} \quad g\left(\frac{1}{4}, 0\right) = \text{lerp}(\langle 2, 0, 0 \rangle, \langle 0, 1, 0 \rangle, \frac{1}{4}) = \left\langle \frac{3}{2}, \frac{1}{4}, 0 \right\rangle$$

$$g\left(\frac{1}{4}, 1\right) = \text{lerp}(\langle 0, 2, 4 \rangle, \langle 2, 2, 0 \rangle, \frac{1}{4}) = \left\langle \frac{1}{2}, 2, 3 \right\rangle.$$

$$g\left(\frac{1}{4}, \frac{1}{2}\right) = \text{lerp}(\langle \frac{3}{2}, \frac{1}{4}, 0 \rangle, \langle \frac{1}{2}, 2, 3 \rangle, \frac{1}{2}) = \left\langle \frac{1}{2}, \frac{9}{8}, \frac{3}{2} \right\rangle.$$

Straightforward differentiation gives the formulas for the partial derivatives.

The crossproduct of the two formulas gives a normal vector. It is not necessary to expand it out. But you do, can find that it is obtainable by bilinear interpolation from the four vectors

$$(\vec{y} - \vec{x}) \times (\vec{w} - \vec{x}), \quad (\vec{z} - \vec{y}) \times (\vec{x} - \vec{y}), \quad (\vec{w} - \vec{z}) \times (\vec{y} - \vec{z})$$

$$\text{and } (\vec{x} - \vec{w}) \times (\vec{z} - \vec{w}).$$

#4: (a)  $\vec{w} = \left\langle \frac{3}{4}, \frac{3}{2} \right\rangle$

$$(b) \frac{1}{4}\vec{x} + \frac{3}{4}\vec{y} = \langle 0, 0, \frac{1}{2} \rangle + \langle 3, 6, 3 \rangle = \langle 3, 6, \frac{7}{2} \rangle$$

$$(c) \text{We know } \alpha = 1 - \beta. \text{ So } \alpha\vec{x} + \beta\vec{y} = (1 - \beta)\vec{x} + \beta\vec{y}$$

$$= \langle 0, 0, 2(1 - \beta) \rangle + \langle 4\beta, 8\beta, 4\beta \rangle$$

$$= \langle 4\beta, 8\beta, 2 + 2\beta \rangle$$

For this to represent  $\vec{w} = \langle \frac{3}{4}, \frac{3}{2} \rangle$ , need  $4\beta = \frac{3}{4}(2 + 2\beta)$   
 i.e.  $16\beta = 6 + 6\beta$  so  $10\beta = 6$  or  $\beta = \frac{3}{5}$ . And  $\alpha = \frac{2}{5}$ .