Math 155A — Computer Graphics — Winter 2019
Homework #7 — Due Wednesday, March 20, 9:00pm
Hand in via Gradescope — Use separate pages for each problem.

1. A particle moving in $\mathbb{R}$ (on a line) has position given by a cubic function $q(t)$. At time $t = 0$ it is at position $q(0) = 1$ and has initial velocity $q'(0) = 3$. At time $t = 1$ it is at $q(1) = 8$ and has velocity $q'(1) = -6$. Express $q(u)$ as a Bézier "curve" in $\mathbb{R}$ by giving its four control points.

The convex hull property can be used to bound the position $q(t)$ for $t \in [0, 1]$. What bounds does this give? (Do not find the actual maximum, just give the upper bound that comes from the convex hull property.)

Express $q(u)$ as a degree three polynomial of t.

2 A degree three Bézier curve in $\mathbb{R}^2$ has control points $\mathbf{p}_0 = \langle 0, 0\rangle$, $\mathbf{p}_1 = \langle 1, -2\rangle$, $\mathbf{p}_2 = \langle 1, 3\rangle$, and $\mathbf{p}_2 = \langle 3, 0\rangle$.

Draw a graph of the curve, and the results of (one round of) recursive subdivision with $u = \frac{1}{2}$. Show your work clearly in your sketch of the curve. What is $q(\frac{1}{2})$? (This is essentially the same as using the de Casteljau algorithm to find $q(\frac{1}{2})$.)

The second half of the curve $q(u)$ where $u \in [\frac{1}{2}, 1]$ is also a degree three Bézier curve. What are its control points?

3. (Catmull-Rom interpolation.) Let $\mathbf{p}_0 = \langle 0, -2\rangle$. $\mathbf{p}_1 = \langle 2, 0\rangle$, $\mathbf{p}_2 = \langle 2, 1\rangle$, $\mathbf{p}_3 = \langle 4, 1\rangle$, and $\mathbf{p}_4 = \langle 4, 0\rangle$.

Sketch the Catmull-Rom curve $\mathbf{q}(u)$ specified by these points. What is its starting point? What is its ending point? What is the slope of $\mathbf{q}(u)$ at each interpolated point $\mathbf{p}_i$?

The subcurve of $\mathbf{q}$ that joins $\mathbf{p}_1$ and $\mathbf{p}_2$ is a degree Bézier curve. What are its control points?