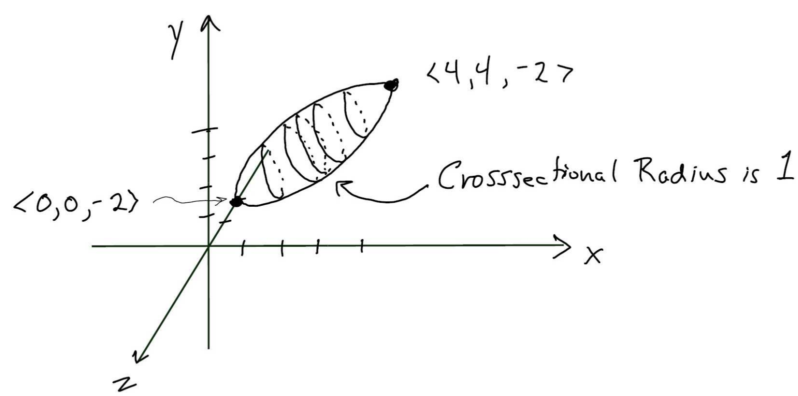
Math 155A — Computer Graphics — Winter 2020
Homework #2 — Due Tuesday, January 28, 9:00pm
Hand in via Gradescope — Use separate pages for each problem 1.-4.

1. Let $S_{a,b,c}$ be the non-uniform scaling transformation $S_{a,b,c}(\langle x_1, x_2, x_3 \rangle) = \langle ax_1, bx_2, cx_3 \rangle$ (in $\mathbb{R}^3$). Express $S_{a,b,c}$ as a 4×4 matrix over homogeneous coordinates.

2. Let $T_{\mathbf{u}}$ and $R_{\theta,\mathbf{u}}$ be the translation and rotation operators in $\mathbb{R}^3$. Give the 4×4 matrix representations of $T_{\mathbf{i}-\mathbf{k}} \circ R_{\pi/2,\mathbf{j}}$ and of $R_{\pi/2,\mathbf{j}} \circ T_{\mathbf{i}-\mathbf{k}}$. (You can do this either by multiplying matrices or by visualizing. If you do it with matrices, you are highly recommended to try visualizing afterwards.)

3. A $2 \times 2 \times 2$ cube has the eight vertices $\langle \pm 1, \pm 1, \pm 1 \rangle$. Show how to render the six faces of the cube with two triangle fans, by explicitly listing the vertices used for the triangle fans. Make sure that the usual CCW front faces are facing outwards. (There is more than one way to do this, but please use $\langle 1, 1, 1 \rangle$ as the first (central) vertex for one of the triangle fans.)

4. As shown in the figure, an elongated ellipsoid in $\mathbb{R}^3$ has one end of its longest axis at $\langle 0, 0, -2 \rangle$ and the other end at $\langle 4, 4, -2 \rangle$. At its center, the ellipsoid has a circular cross section of radius 1. The center is positioned at $\langle 2, 2, -2 \rangle$; the ellipsoid is slanted at a angle of $\pi/4$ (45°).



Suppose $\mathcal{S}$ is a sphere of radius one centered at the origin. Both questions (a) and (b) below ask for an affine transformation f such that $f(\mathcal{S})$ is the elongated ellipsoid as pictured above.

(a) Give a 4×4 matrix which which represents f (by acting on homogeneous representations of points in $\mathbb{R}^3$).

(b) Express f as a composition of rotations $R_{\theta,\mathbf{u}}$, translations $T_{\mathbf{u}}$ and scaling transformations $S_{a,b,c}$.