Math 155A — Computer Graphics — Winter 2020
Homework #3 — Due Tuesday, February 4, 9:00pm
Hand in via Gradescope — Use separate pages for each problem 1.-3.

1. Suppose $\mathcal{C}$ is a radius 1, height 2 cylinder centered at the origin, with central axis the y-axis. The top face of $\mathcal{C}$ is the horizontal disk of radius one centered $\langle 0,1,0\rangle$. The bottom face of $\mathcal{C}$ is the horizontal disk of radius one centered $\langle 0,-1,0\rangle$. ("Horizontal" means parallel to the xz-plane.)

Let $\mathcal{D}$ a skewed cylinder (also called an oblique cylinder, please "google" it) which has central axis the line where $y = x+2$ and $z = 0$, and has perpendicular height 4, with the top face of $\mathcal{D}$ the horizontal radius $\frac{1}{2}$ disk centered at $\langle 2,4,0\rangle$ and the bottom face of $\mathcal{D}$ the horizontal radius $\frac{1}{2}$ disk centered at $\langle -2,0,0\rangle$.

a. Draw a picture of D. What is the center point of D?

b. Give a 4×4 matrix M so that M represents the affine transformation sending $\mathcal{C}$ to $\mathcal{D}$.

2. Let B be the transformation $\langle x,y,z\rangle \mapsto \langle \frac{1+x}{1-y} - 1,\, 0,\, \frac{z}{1-y}\rangle$. Give a 4×4 matrix which represents this transformation over homogeneous coordinates. (When $0 < y < 1$, B gives the transformation for a shadow cast from a light at $\langle -1,1,0\rangle$ onto the plane $y = 0$. You do **not** need to use this fact to work the problem!)

3. A light source is placed at $\langle -10,1,0\rangle$ and it casts shadows onto the yz-plane P defined by $x = 2$. The $x = 2$ plane is parallel to the yz-plane and acts like an infinite wall.

When $\langle x,y,z\rangle$ is a point in $\mathbb{R}^3$ with $-10<x\leq 2$, define $A(\langle x,y,z\rangle)$ to be the position of the shadow of the point on the yz-plane. For example, $A(\langle -4,2,2\rangle) = \langle 2,3,4\rangle$, and $A(\langle -7,2,2\rangle) = \langle 2,5,8\rangle$.

a. Working in ordinary coordinates (not homogeneous) give the formula expressing the mapping $A(\langle x,y,z\rangle) = \langle x',y',z'\rangle$. That is, give formulas for x',y',z' in terms of x,y,z.

b. Give a 4×4-matrix that represents the transformation A over homogeneous coordinates.