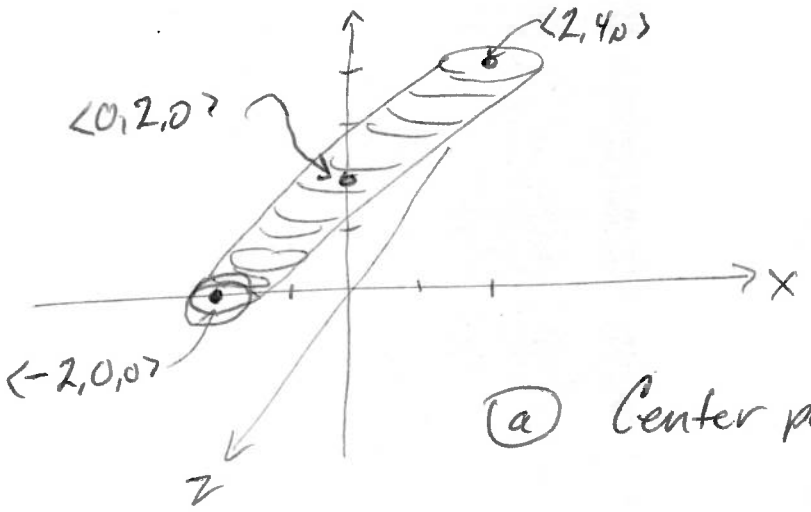
Math 155A - Homework #3 Solutions - Winter 2020

1/



Horizontal sections are circles of radius $\frac{1}{2}$

(a) Center point is $\langle 0,2,0 \rangle$

(b) 4×4 matrix:

$$\begin{pmatrix} \frac{1}{2} & 2 & 0 & 0 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(Other answers are also possible.)

2/ $\langle x,y,z \rangle \longmapsto \left\langle \frac{1+x}{1-y} - 1,\ 0,\ \frac{z}{1-y} \right\rangle$

In homogeneous coordinates:

$$\langle x,y,z,1 \rangle \longmapsto \left\langle \frac{1+x}{1-y} - 1,\ 0,\ \frac{z}{1-y},\ 1 \right\rangle$$

$$\equiv \langle 1+x-(1-y),\ 0,\ z,\ 1-y \rangle$$

$$= \langle x+y,\ 0,\ z,\ 1-y \rangle.$$

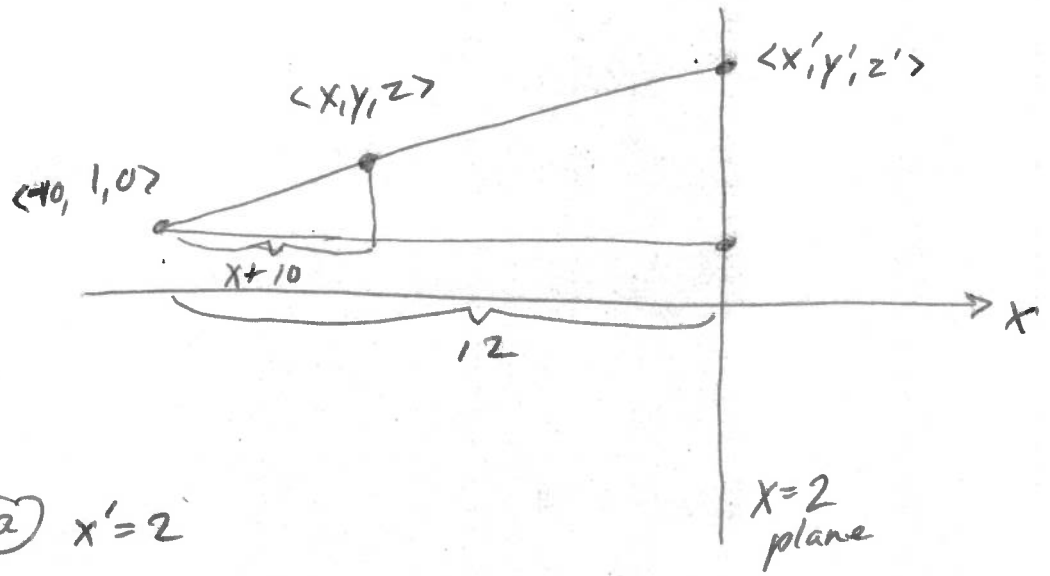
More generally:

$$\langle x,y,z,w \rangle \longmapsto \langle x+y,\ 0,\ z,\ w-y \rangle$$

4×4 matrix:

$$\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix}$$

HW 3 answers – continued



(a) $x' = 2$

$$\frac{y-1}{x+10} = \frac{y'-1}{12} \quad \text{so} \quad y'-1 = \frac{12y-12}{x+10}$$

$$y' = \frac{12y-12}{x+10} + 1$$

$$\frac{z}{x+10} = \frac{z'}{12} \quad \text{so} \quad z' = \frac{12z}{x+10}$$

$$\langle x, y, z \rangle \longmapsto \left\langle 2, \frac{12y-12}{x+10} + 1, \frac{12z}{x+10} \right\rangle$$

(b) In homogeneous coordinates:

$$\langle x, y, z, 1 \rangle \mapsto \left\langle 2, \frac{12y-12}{x+10} + 1, \frac{12z}{x+10}, 1 \right\rangle$$

$$\equiv \langle 2x+20, 12y-12+x+10, 12z, x+10 \rangle$$

$$= \langle 2x+20, x+12y-2, 12z, x+10 \rangle$$

Alternately:

$$\langle x, y, z, w \rangle \mapsto \langle 2x+20w, x+12y-2w, 12z, x+10w \rangle$$

4×4 matrix:

$$\begin{pmatrix} 2 & 0 & 0 & 20 \\ 1 & 12 & 0 & -2 \\ 0 & 0 & 12 & 0 \\ 1 & 0 & 0 & 10 \end{pmatrix}$$