Math 155A — Computer Graphics — Winter 2020
Homework #4 — Due Monday, February 24, 9:00pm
Hand in via Gradescope — Use separate pages for each problem.

1. Let $h(r) = \sqrt{1+r^2}$ define a surface of revolution $\mathbf{f}(r,\theta)$ by revolving the graph of h around the y-axis. Give the formula for a point $\mathbf{f}(\theta, r)$ on the surface of rotation. Also give a formula for a normal vector (not necessarily a unit vector) at that point on the surface of rotation. Your normal vector should be pointing generally upward. Your answers should be stated as functions of r and θ.

2. Let S be the hyperboloid surface defined as a level surface

$$S \;=\; \{\langle x, y, z\rangle : y = \sqrt{1+x^2+z^2}\}. \tag{1}$$

Use the level set surface (gradient) method to give a formula $\mathbf{n} = \mathbf{n}(x,y,z)$ for a vector normal to S at a point $\langle x, y, z\rangle$ on S. [Hint: it will be easier if you use the equivalent definition: $S = \{\langle x, y, z\rangle : y^2 = 1 + x^2 + z^2,\ y \geq 0\}$. The vector $\mathbf{n}$ should be pointing generally upward. Your answer should be given as a function of x, y, z.

The surface S is the same as the surface of rotation in Problem 1. Does your answer agree with the answer to Problem 1? If so, how? If not, why not?

3. Let S be the hyperboloid surface (1) of the previous exercise. Let $A : \mathbb{R}^3 \to \mathbb{R}^3$ be the **linear** transformation with 3×3 matrix representation

$$M \;=\; \begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Note that A is a shearing. What is $(M^{-1})^T$? Suppose $\langle x, y, z\rangle$ is a point on the surface S. Its normal vector $\mathbf{n}(x,y,z)$ was already computed in the previous exercise. Give a formula for a vector $\mathbf{m} = \mathbf{m}(x,y,z)$ normal to the transformed surface $A(S)$ at the transformed point $A(\langle x, y, z\rangle)$. m does not need to be a unit vector.

4. Let S' be the surface defined be $y = 3 + x - x^2 z$. For $\langle x, y, z\rangle$ a point on S', give a formula for a vector normal to the surface at $\langle x, y, z\rangle$. Orient the vector so it is point upward (positive y component).

You may use either the crossproduct of partial derivatives method, or the gradient (level set) method. (Hand in only one solution, but you should know how to do both methods!)