1. Let \( h(r) = \sqrt{1 + r^2} \) define a surface of revolution \( f(r, \theta) \) by revolving the graph of \( h \) around the \( y \)-axis. Give the formula for a point \( f(\theta, r) \) on the surface of rotation. Also give a formula for a normal vector (not necessarily a unit vector) at that point on the surface of rotation. Your normal vector should be pointing generally upward. Your answers should be stated as functions of \( r \) and \( \theta \).

2. Let \( S \) be the hyperboloid surface defined as a level surface

\[
S = \{ (x, y, z) : y = \sqrt{1 + x^2 + z^2} \}. \tag{1}
\]

Use the level set surface (gradient) method to give a formula \( n = n(x, y, z) \) for a vector normal to \( S \) at a point \( (x, y, z) \) on \( S \). [Hint: it will be easier if you use the equivalent definition: \( S = \{ (x, y, z) : y^2 = 1 + x^2 + z^2, \ y \geq 0 \} \). The vector \( n \) should be pointing generally upward. Your answer should be given as a function of \( x, y, z \).

The surface \( S \) is the same as the surface of rotation in Problem 1. Does your answer agree with the answer to Problem 1? If so, how? If not, why not?

3. Let \( S \) be the hyperboloid surface (1) of the previous exercise. Let \( A : \mathbb{R}^3 \to \mathbb{R}^3 \) be the linear transformation with \( 3 \times 3 \) matrix representation

\[
M = \begin{pmatrix}
1 & 0 & -2 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix}.
\]

Note that \( A \) is a shearing. What is \((M^{-1})^T\)? Suppose \( (x, y, z) \) is a point on the surface \( S \). Its normal vector \( n(x, y, z) \) was already computed in the previous exercise. Give a formula for a vector \( m = m(x, y, z) \) normal to the transformed surface \( A(S) \) at the transformed point \( A((x, y, z)) \). \( m \) does not need to be a unit vector.

4. Let \( S' \) be the surface defined be \( y = 3 + x - x^2 z \). For \( (x, y, z) \) a point on \( S' \), give a formula for a vector normal to the surface at \( (x, y, z) \). Orient the vector so it is point upward (positive \( y \) component).

You may use either the crossproduct of partial derivatives method, or the gradient (level set) method. (Hand in only one solution, but you should know how to do both methods!)