Math 155A – Homework #5 – Selected Answers, p1

1. $\vec{x}_1 = \langle -2, 0\rangle \quad \vec{x}_2 = \langle 4, 1\rangle$

(a) $Lerp(\vec{x}_1, \vec{x}_2, \alpha) = ?$

$\alpha = -2: \quad \langle -14, -2\rangle$

$\alpha = -1: \quad \langle -8, -1\rangle$

$\alpha = 0: \quad \langle -2, 0\rangle$

$\alpha = 1/10: \quad \langle -1.4, 1/10\rangle$

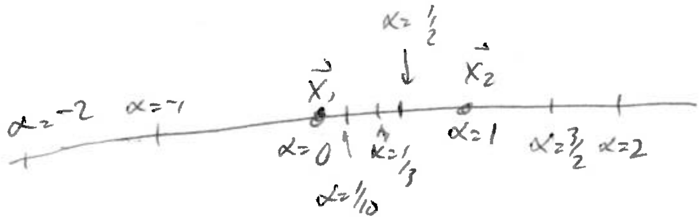
$\alpha = 1/3: \quad \langle 0, 1/3\rangle$

$\alpha = \frac{1}{2}: \quad \langle 1, 1/2\rangle$

$\alpha = 1: \quad \langle 4, 1\rangle$

$\alpha = 1\frac{1}{2}: \quad \langle 7, 1\frac{1}{2}\rangle$

$\alpha = 2: \quad \langle 10, 2\rangle$



$\alpha = 2/3$ gives $\langle 2, 2/3\rangle$

$\alpha = 3$ gives $\langle 16, 3\rangle$

(b) ~~$\vec{u} =$~~ $$\frac{(\vec{0} - \vec{x}_1)\cdot(\vec{x}_2 - \vec{x}_1)}{(\vec{x}_2 - \vec{x}_1)^2} = \frac{\langle 2, 0\rangle \cdot \langle 6, 1\rangle}{\|\langle 6, 1\rangle\|^2} = \frac{12}{37}$$

$\vec{u} = lerp(\vec{x}_1, \vec{x}_2, \frac{12}{37})$

(c) $\langle 2, 2/3\rangle = lerp(\vec{x}_1, \vec{x}_2, 2/3)$

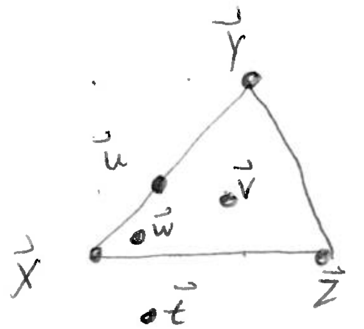
So $f(\langle 2, 2/3\rangle) = lerp(f(\vec{x}_1), f(\vec{x}_2), 2/3) = lerp(-3, 3, 2/3) = 1$

2. (a) lerp($\vec{x}$, $\vec{y}$, α) is a linear combination of $\vec{x}$, $\vec{y}$ for all $\alpha \in \mathbb{R}$

(b) It is an affine combination for all α

(c) It is a weighted average of x, and y for $0 \le \alpha \le 1$.

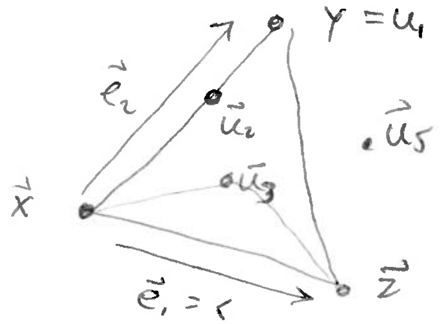
3 $\vec{x} = \langle 0, 1 \rangle$, $\vec{y} = \langle 2, 3 \rangle$, $\vec{z} = \langle 3, 0 \rangle$



$\vec{t}$ is farther below the triangle than is shown in the figure.

	α	β	γ	$\alpha\vec{x} + \beta\vec{y} + \gamma\vec{z}$
(a)	0	1	0	$\vec{y} = \langle 2, 3 \rangle$
(b)	$2/3$	$1/3$	0	$\langle \frac{2}{3}, \frac{5}{3} \rangle =: \vec{u}$
(c)	$1/3$	$1/3$	$1/3$	$\langle \frac{5}{3}, \frac{4}{3} \rangle =: \vec{v}$
(d)	$4/5$	$\frac{1}{10}$	$\frac{1}{10}$	$\langle \frac{1}{2}, \frac{11}{5} \rangle =: \vec{w}$
(e)	$4/3$	$\frac{2}{3}$	-1	$\langle -\frac{5}{3}, \frac{10}{3} \rangle =: \vec{t}$

HW #5 - Selected Answers, p.3



(4.) $\vec{x} = \langle 0,1\rangle$ $\quad \vec{y} = \langle 2,3\rangle$ $\quad \vec{z} = \langle 3,0\rangle$

$\vec{e}_1 = \vec{z} - \vec{x} = \langle 3,-1\rangle$

$\vec{e}_2 = \vec{y} - \vec{x} = \langle 2,2\rangle$

(a) $\vec{u}_1 = \langle 2,3\rangle = \vec{y}$

$\alpha = 0,\ \beta = 1,\ \gamma = 0$

(b) $\vec{u}_2 = \langle \frac{4}{3}, \frac{7}{3}\rangle = \langle 1\tfrac{1}{3}, 2\tfrac{1}{3}\rangle$

From the picture $\vec{u}_2$ is on the edge $\overline{\vec{x}\vec{y}}$ and by inspection $\boxed{\alpha = 1/3 \quad \beta = 2/3,\ \gamma = 0}$

(c) $\vec{u}_3 = \langle 3/2, 3/2\rangle$

$\vec{f} = \vec{u}_3 - \vec{x} = \langle 3/2, 1/2\rangle$

$2D = |e_1\, e_2| = \begin{vmatrix} 3 & 2 \\ -1 & 2 \end{vmatrix} = 8$

$2B = |e_1 f| = \begin{vmatrix} 3 & 3/2 \\ -1 & 1/2 \end{vmatrix} = \frac{3}{2} + \frac{3}{2} = 3$

$\beta = 3/8 \quad (= B/D)$

$2C = |f\, e_2| = \begin{vmatrix} 3/2 & 2 \\ 1/2 & 2 \end{vmatrix} = 3 - 1 = 2$

$\gamma = 2/8 = 1/4 \quad (= C/D)$

So $\boxed{\alpha = \frac{3}{8} \quad \beta = 3/8,\ \gamma = 1/4.}$

(d) $\vec{u}_4 = \langle 1,0\rangle$. Now $\vec{f} = \vec{u}_4 - \vec{x} = \langle 1,-1\rangle$

$2B = |e_1 f| = \begin{vmatrix} 3 & 1 \\ -1 & -1 \end{vmatrix} = -3 + 1 = -2$ (negative area - formulas still work!)

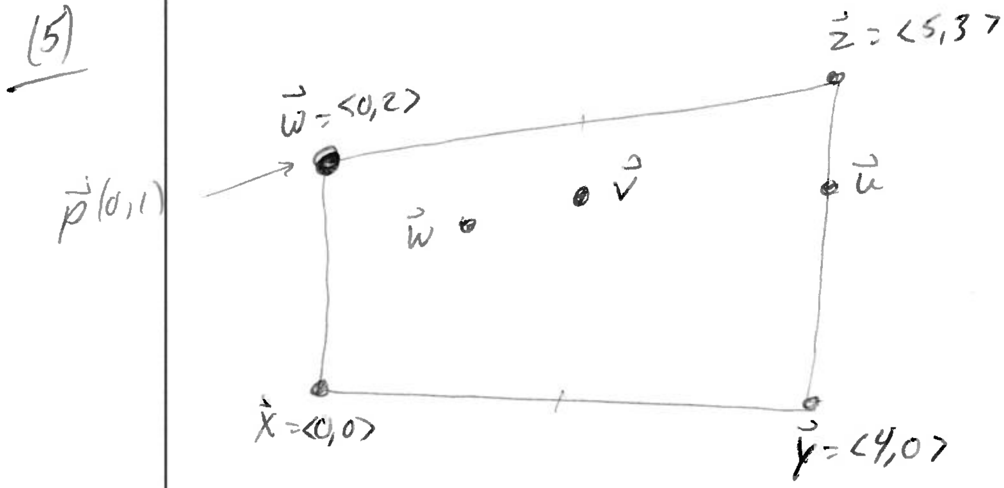
$2C = |f e_2| = \begin{vmatrix} 1 & 2 \\ -1 & 2 \end{vmatrix} = 2 + 2 = 4$

$\boxed{\beta = -2/8 = -1/4 \quad \gamma = 4/8 = 1/2 \quad \alpha = 1 - \beta - \gamma = 3/4}$

(e) For $\vec{u}_5$, $\alpha < 0$, $\beta > 0$, and $\gamma > 0$.

HW #5 - Selected Answers, p.4

(5)



(a) $\vec{p}(0,1) = \vec{w} = \langle 0,2 \rangle$

(b) $\vec{p}(\frac{2}{3},1) = \langle 4\frac{2}{3}, 2 \rangle =: \vec{u}$

(c) $\vec{p}(\frac{1}{2},\frac{3}{4}) = \langle 2\frac{1}{4}, 1\frac{7}{8} \rangle =: \vec{v}$

(d) $\vec{p}(\frac{1}{3},\frac{2}{3}) = \langle \frac{14}{9}, \frac{14}{9} \rangle =: \vec{w}$

6 $\vec{u}(\alpha,\beta) = (1-\alpha)[(1-\beta)\vec{x} + \beta\vec{w}] + \alpha[(1-\beta)\vec{y} + \beta\vec{z}]$

$\frac{\partial \vec{u}}{\partial \alpha} = ((1-\beta)(\vec{y}-x) + \beta(\vec{z}-\vec{w})) = \text{lerp}(\vec{y}-\vec{x}, \vec{z}-\vec{w}, \beta)$

$\frac{\partial \vec{u}}{\partial \beta} = (1-\alpha)(\vec{w}-\vec{x}) + \alpha(\vec{z}-\vec{y}) = \text{lerp}(\vec{w}-\vec{x}, \vec{z}-\vec{y}, \alpha)$

$\frac{\partial \vec{u}}{\partial \alpha} \times \frac{\partial \vec{u}}{\partial \beta}$ gives a normal vector (if non-zero)

A further result:

$$\frac{\partial \vec{u}}{\partial \alpha} \times \frac{\partial \vec{u}}{\partial \beta} = (1-\alpha)(1-\beta)((\vec{y}-\vec{x}) \times (\vec{w}-\vec{x})) + \alpha(1-\beta)((\vec{y}-\vec{x}) \times (\vec{z}-\vec{y})) + (1-\alpha)\beta((\vec{z}-\vec{w}) \times (\vec{w}-\vec{x})) + \alpha\beta((\vec{z}-\vec{w}) \times (\vec{z}-\vec{x}))$$

i.e. $\frac{\partial \vec{u}}{\partial \alpha} \times \frac{\partial \vec{u}}{\partial \beta}$ is obtained from the four normals at the corners by bilinear interpolation!