Math 155B - Intro to Computer Graphics II

Midterm Examination

February 20, 2004

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Write your name or initials on every page before beginning the exam.

You have 50 minutes. There are five problems. You may use a calculator (not programmed with spline functions, please!) and the supplied cheat sheet. You may not use your own notes, or the textbook or other materials. You must show your work in order to get credit. Good luck!

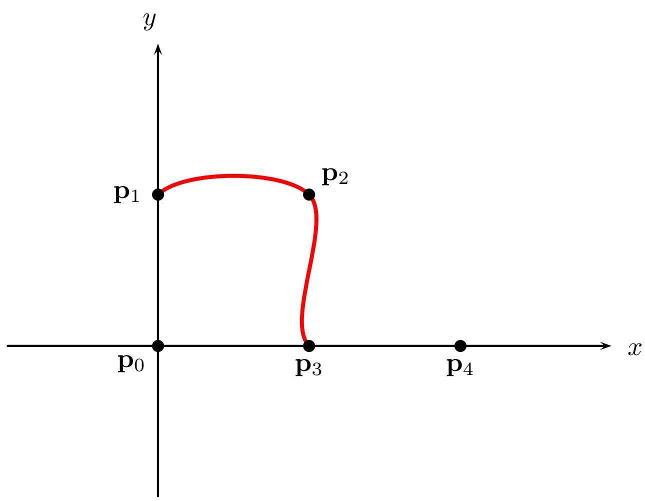
Name: Answer Key

Student ID:

1	
2	
3	
4	
5	
Total	

1. Let the Catmull-Rom curve $\mathbf{q}(u)$ be defined by the following control points:

$$\begin{aligned} \mathbf{p}_0 &= \langle 0, 0 \rangle \\ \mathbf{p}_1 &= \langle 0, 1 \rangle \\ \mathbf{p}_2 &= \langle 1, 1 \rangle \\ \mathbf{p}_3 &= \langle 1, 0 \rangle \\ \mathbf{p}_4 &= \langle 2, 0 \rangle \end{aligned}$$



Thus, $\mathbf{q}(i) = \mathbf{p}_i$ for $i = 1, 2, 3$. For the problems below, show and label all your work, esp. for part d.

a. Give a freehand sketch of the Catmull Rom curve $\mathbf{q}(u)$ on the graph below.

b. What is the value of $\mathbf{q}'(1)$?

c. What is the value of $\mathbf{q}'(2)$?

d. What is the value of $\mathbf{q}(\frac{3}{2})$?

<u>ANSWERS:</u>

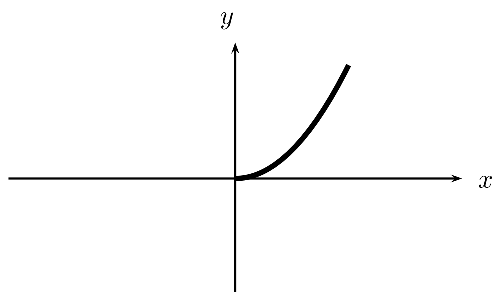
b. $\mathbf{q}'(1) \;=\; \langle \frac{1}{2}, \frac{1}{2} \rangle$.

c. $\mathbf{q}'(2) \;=\; \langle \frac{1}{2}, -\frac{1}{2} \rangle$.

d. Partial credit for: $\mathbf{p}_1^+ = \langle \frac{1}{6}, \frac{7}{6} \rangle$ and $\mathbf{p}_2^- = \langle \frac{5}{6}, \frac{7}{6} \rangle$;

$$\text{Answer:} \quad \mathbf{q}(\frac{3}{2}) \;=\; \langle \frac{1}{2}, \frac{9}{8} \rangle.$$

2. Let $\mathbf{q}(u)$ be the quadratic curve defined by $\mathbf{q}(u) = \langle u, u^2 \rangle$ for $0 \leqslant u \leqslant 1$. Note that $\mathbf{q}(u)$ traces out a portion of a parabola.



a. Express $\mathbf{q}(u)$ as a degree 2 Bezier curve, by giving its control points. Use a rational curve if necessary.

b. Express $\mathbf{q}(u)$ as a degree 3 Bezier curve, by giving its control points. Again, use a rational curve if necessary.

<u>ANSWERS:</u>

a. $\mathbf{p}_0 = \langle 0, 0 \rangle$, $\mathbf{p}_1 = \langle \frac{1}{2}, 0 \rangle$, $\mathbf{p}_2 = \langle 1, 1 \rangle$ defines $\mathbf{q}(u)$ as a degree 2 Bézier curve.

b. $\hat{\mathbf{p}}_0 = \langle 0, 0 \rangle$, $\hat{\mathbf{p}}_1 = \langle \frac{1}{3}, 0 \rangle$, $\hat{\mathbf{p}}_2 = \langle \frac{2}{3}, \frac{1}{3} \rangle$, $\hat{\mathbf{p}}_3 = \langle 1, 1 \rangle$ defines $\mathbf{q}(u)$ as a degree 3 Bézier curve.

3. Describe what knot insertion does to a curve. Also describe (briefly!) some possible applications for knot insertion. The more applications you can give, the better.

ANSWERS: Knot insertion does not change the curve.

Applications:
- Extra knots give the designer more ways to edit the curve shape.
- Allow the designer to modify the curve to have "corners" or other loss of higher-order continuity.
- To refine two different knot vectors to become equal.
- To convert a B-spline curve into a piecewise Bezier curve.
- A first step in degree elevation.
- To make the control polygon more closely approximate the B-spline curve. For example, to exploit convex hull properties.

4. A B-spline curve $\mathbf{q}(u)$ of order 3 has knot vector

$$[0, 0, 0, 1, 2, 2, 3, 4, 5, 6, 7].$$

a. How many control points does this curve have?
ANS: 8 control points.

b. For what values of i must the curve be C^i-continuous at $u = 2$?
ANS: $i = 0$

c. For what values of i must the curve be C^i-continuous at $u = 3$?
ANS: $i = 0, 1$.

d. For what values of i must the curve be C^i-continuous at $u = 2.5$?
ANS: $i = 0, 1, 2, 3, \ldots$ (all non-negative integers).

e. What is the domain of the curve? (I.e., for what values of u is $\mathbf{q}(u)$ defined?)
ANS: The domain is $[0, 5]$.

5. The front half of a cylinder of radius 2 consists of the points

$$\{\langle x, y, z\rangle : x^2 + z^2 = 4 \text{ and } 0 \leqslant y \leqslant 1 \text{ and } z \geqslant 0\}$$

Express this surface as a single Bezier patch (of any order you wish). Define the surface by giving its control points. You should use a rational surface if necessary.

ONE POSSIBLE ANSWER:

For an order 2×3 Bézier patch, use the control points

$$\begin{array}{lll} \mathbf{p}_{1,0} = \langle -2, 1, 0, 1\rangle & \mathbf{p}_{1,1} = \langle 0, 0, 2, 0\rangle & \mathbf{p}_{1,2} = \langle 2, 1, 0, 1\rangle \\ \mathbf{p}_{0,0} = \langle -2, 0, 0, 1\rangle & \mathbf{p}_{0,1} = \langle 0, 0, 2, 0\rangle & \mathbf{p}_{0,2} = \langle 2, 0, 0, 1\rangle \end{array}$$