Math 155B - Homework #1 - Answers to Selected Problems - Sam Buss

Winter 2005

1: Given $\vec{p}_0 = \langle 0,0\rangle$, $\vec{p}_1 = \langle 3,3\rangle$, $\vec{p}_2 = \langle 3,0\rangle$.

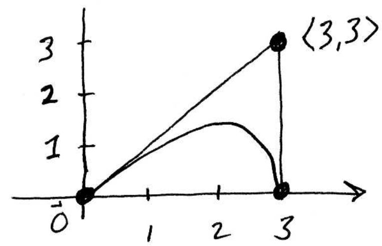
(a) $\vec{r}_0 = \frac{1}{2}(\vec{p}_0 + \vec{p}_1) = \langle \frac{3}{2}, \frac{3}{2}\rangle$;

$\vec{r}_1 = \frac{1}{2}(\vec{p}_0 + \vec{p}_1) = \langle 3, 3/2\rangle$

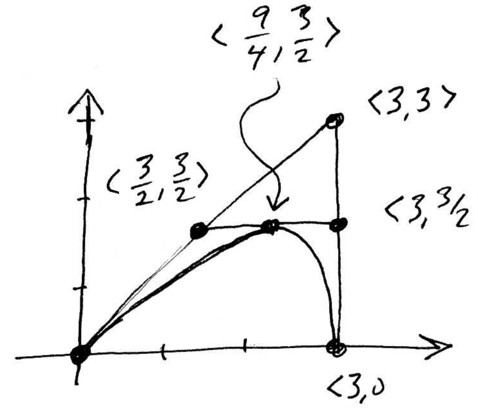
$\vec{q}(1/2) = \vec{s}_0 = \frac{1}{2}(\vec{r}_0 + \vec{r}_1) = \langle \frac{9}{4}, \frac{3}{2}\rangle$

(d) For $u = 1/3$;
$\vec{q}(1/3) = \langle 5/3, 4/3\rangle$

(b)



(c):



(c): Control points for 1^{st} curve: $\langle 0,0\rangle$, $\langle 3/2, 3/2\rangle$, $\langle 9/4, 3/2\rangle$

Control points for 2^{nd} curve: $\langle 9/4, 3/2\rangle$, $\langle 3, 3/2\rangle$, $\langle 3,0\rangle$

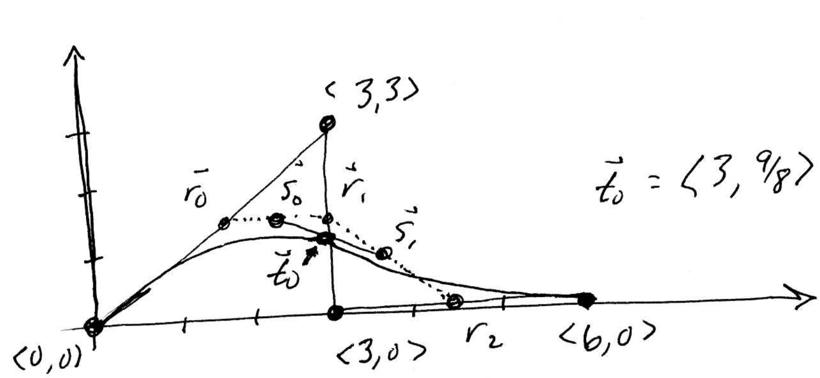
2: Given $\vec{p}_0 = \langle 0,0\rangle$, $\vec{p}_1 = \langle 3,3\rangle$, $\vec{p}_2 = \langle 3,0\rangle$, $\vec{p}_3 = \langle 6,0\rangle$.

(a) $\vec{r}_0 = \frac{1}{2}(\vec{p}_0 + \vec{p}_1) = \langle \frac{3}{2}, \frac{3}{2}\rangle$ $\quad \vec{r}_1 = \frac{1}{2}(\vec{p}_1 + \vec{p}_2) = \langle 3, 3/2\rangle$, $\quad \vec{r}_2 = \frac{1}{2}(\vec{p}_2 + \vec{p}_3) = \langle 4\frac{1}{2}, 0\rangle$

$\vec{s}_0 = \frac{1}{2}(\vec{r}_0 + \vec{r}_1) = \langle \frac{9}{4}, 3/2\rangle$ $\quad \vec{s}_1 = \frac{1}{2}(\vec{r}_1 + \vec{r}_2) = \langle \frac{15}{4}, \frac{3}{4}\rangle$

$\vec{q}(1/2) = \vec{t}_0 = \frac{1}{2}(\vec{s}_0 + \vec{s}_1) = (3, \frac{9}{8})$

(b)



$\vec{t}_0 = \langle 3, 9/8\rangle = \vec{q}(1/2)$

(d) For $u = 1/3$:

$\vec{r}_0 = \frac{2}{3}\vec{p}_0 + \frac{1}{3}\vec{p}_1 = \langle 1,1\rangle$ $\quad \vec{r}_1 = \frac{2}{3}\vec{p}_1 + \frac{1}{3}\vec{p}_2 = \langle 3,2\rangle$, $\quad \vec{r}_2 = \frac{2}{3}s_2 + \frac{1}{3}s_3 = \langle 4,0\rangle$

$\vec{s}_0 = \frac{2}{3}\vec{r}_0 + \frac{1}{3}\vec{r}_1 = \langle \frac{5}{3}, \frac{4}{3}\rangle$ $\quad \vec{s}_1 = \frac{2}{3}\vec{r}_1 + \frac{1}{3}\vec{r}_2 = \langle \frac{10}{3}, \frac{4}{3}\rangle$

$\vec{t}_0 = \frac{2}{3}\vec{s}_0 + \frac{1}{3}\vec{s}_1 = \langle \frac{20}{9}, \frac{4}{3}\rangle$.

3: Control points as a degree curve are:

$\langle 0,0\rangle$, $\langle 2,2\rangle$, $\langle 3,2\rangle$, $\langle 3,0\rangle$.

4: Note that $g(0)=1$, $g(1)=3$, $g'(0)=g'(1)=2$.

Thus $p_0=1$, $p_1=1+\frac{2}{3}=\frac{5}{3}$, $p_2=3-\frac{2}{3}=\frac{7}{3}$, $p_3=3$.

That is, the control points are $1, 5/3, 7/3, 3$.

5 (a) To get $\theta(0)=30$, $\theta'(0)=30$, $\theta(1)=35$, $\theta'(1)=0$, use

$$\theta(u) = 30\cdot H_0(u) + 30\cdot H_1(u) + 0\cdot H_2(u) + 35\cdot H_3(u)$$
$$= 30\cdot H_0(u) + 30\cdot H_1(u) + 35\cdot H_3(u).$$

(b) As a Bézier curve its control points are:

$p_0 = 30$

$p_1 = 30 + \frac{30}{3} = 40$

$p_2 = 35 - \frac{0}{3} = 35$

$p_3 = 35$

As in problem 4, this is a a scalar-valued Bézier "curve".

(c) Control points for $\theta'(u)$ as a degree 2 Bézier curve are:

$\vec{r}_0 = 2(p_1-p_0) = 20$

$\vec{r}_1 = 2(p_2-p_1) = -10$

$\vec{r}_3 = 2(p_3-p_2) = 0$

(d) The value of $\theta'(u)$, for $0 \le u \le 1$, is in the convex hull of $\{20,-10,0\}$. Thus $-10 \le \theta'(u) \le 20$ for $0 \le u \le 1$.

Homework #1 - Selected Answers - p3

6 : Exercise VII.24 on p 191. <u>Overhauser part</u>

$\vec{p}_0 = \vec{p}_1 = \langle 0,0\rangle$. $\quad p_2 = \langle 10,0\rangle, \quad p_3 = p_4 = \langle 10,1\rangle.$

$$v_{1/2} = (p_1 - p_0)/(1-0) = \langle 0,0\rangle$$

$$v_{1\frac12} = (p_2-p_1)/(2-1) = \langle 10,0\rangle \Rightarrow \vec{v}_1 = \frac{(2-1)v_{\frac12} + (1-0)v_{1\frac12}}{2-0} = \langle 5,0\rangle$$

$$v_{2\frac12} = (p_3-p_2)/(2.1-2.0) = \langle 0,10\rangle \Rightarrow \vec{v}_2 = \frac{(2.1-2)v_{1\frac12} + (2-1)v_{2\frac12}}{2.1-1} = \frac{\langle 1,10\rangle}{1.1} = \langle \tfrac{10}{11}, \tfrac{100}{11}\rangle$$

$$v_{3\frac12} = (p_4-p_3)/(3.1-2.1) = \langle 0,0\rangle \Rightarrow \vec{v}_3 = \frac{(3.1-2.1)v_{2\frac12} - (2.1-2)v_{3\frac12}}{3.1-2} = \frac{\langle 0,10\rangle}{1.1} = \langle 0, \tfrac{100}{11}\rangle$$

$$p_1^+ = p_1 + \tfrac13(2-1)\vec{v}_1 = \langle 5/3, 0\rangle$$

$$p_2^- = \vec{p}_2 - \tfrac13(2-1)\vec{v}_2 = \langle 10,0\rangle - \langle \tfrac{10}{33}, \tfrac{100}{33}\rangle = \langle 9\tfrac{23}{33}, -3\tfrac{1}{33}\rangle$$

$$p_2^+ = p_2 + \tfrac13(2.1-2)\vec{v}_2 = \langle 10,0\rangle + \langle \tfrac{1}{33}, \tfrac{10}{33}\rangle = \langle 10\tfrac{1}{33}, \tfrac{10}{33}\rangle$$

$$p_3^- = p_3 - \tfrac13(2.1-2)\vec{v}_3 = \langle 10, 1-\tfrac{10}{33}\rangle = \langle 10, \tfrac{23}{33}\rangle$$

<u>Catmull-Rom version</u>

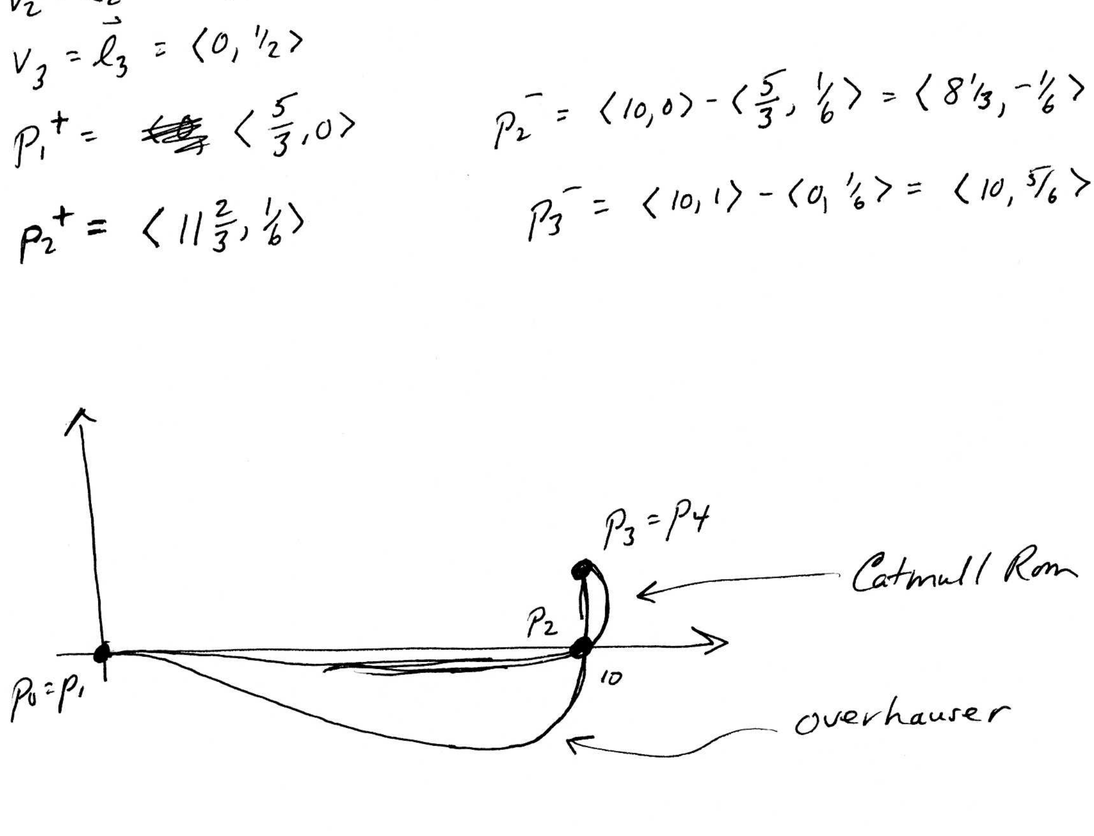
$$\vec{v}_1 = \vec{\ell}_1 = \langle 5,0\rangle$$

$$\vec{v}_2 = \vec{\ell}_2 = \langle 5, \tfrac12\rangle$$

$$v_3 = \vec{\ell}_3 = \langle 0, \tfrac12\rangle$$

$p_1^+ = \langle \tfrac53, 0\rangle \qquad p_2^- = \langle 10,0\rangle - \langle \tfrac53, \tfrac16\rangle = \langle 8\tfrac13, -\tfrac16\rangle$

$p_2^+ = \langle 11\tfrac23, \tfrac16\rangle \qquad p_3^- = \langle 10,1\rangle - \langle 0, \tfrac16\rangle = \langle 10, \tfrac56\rangle$



Homework #1 - Answers to Selected Problems, p4

7: General idea:

Express $\vec{q}(u) = \langle x(u), y(u), w(u) \rangle$ and then show

$$(x(u))^2 + (y(u))^2 = (w(u))^2.$$

Answer omitted

8: We have $\vec{p}_0 = \langle 0,1,1 \rangle$ $\quad \vec{p}_1 = \langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \rangle$, $\vec{p}_2 = \langle 1, 0, 1 \rangle$

Degree elevation gives

$\hat{p}_0 = \vec{p}_0 = \langle 0,1,1 \rangle$, i.e. $\langle 0,1 \rangle$ with weight 1

$$\hat{p}_1 = \frac{1}{3}\vec{p}_0 + \frac{2}{3}\vec{p}_1 = \left\langle \frac{\sqrt{2}}{3}, \frac{1}{3} + \frac{\sqrt{2}}{3}, \frac{1}{3} + \frac{\sqrt{2}}{3} \right\rangle$$

i.e., $\left\langle \frac{\sqrt{2}}{1+\sqrt{2}}, 1 \right\rangle$ with weight $\frac{1+\sqrt{2}}{3}$.

Similar calculation gives

$\hat{p}_2 = \left\langle \frac{1+\sqrt{2}}{3}, \frac{\sqrt{2}}{3}, \frac{1+\sqrt{2}}{3} \right\rangle$, i.e., $\left\langle 1, \frac{\sqrt{2}}{1+\sqrt{2}} \right\rangle$ with weight $\frac{1+\sqrt{2}}{3}$

$\hat{p}_3 = p_2 = \langle 1, 0 \rangle$.