Please show work and label answers clearly.

1. Consider a degree 3 (order 4) B-spline curve $q(u)$ defined with the knot vector 
   $[0, 0, 1, 2, 3, 3, 4, 5, 5, 5, 6, 7, 8, 9, 10]$.
   a. How many control points $p_0, p_1, \ldots$ are needed to define the curve $q(u)$?
   b. What is the domain of $q(u)$?
   c. What is the domain of $N_{0,4}(u)$?
   d. What is the domain of $N_{1,4}(u)$?
   e. What is the domain of $N_{2,4}(u)$?
   f. At $u = 2$, $q(u)$ must be $C^\ell$-continuous, for what value $\ell$?
   g. At $u = 3$, $q(u)$ must be $C^\ell$-continuous, for what value $\ell$?
   h. At $u = 5$, $q(u)$ must be $C^\ell$-continuous, for what value $\ell$?
   i. At $u = 5.5$, $q(u)$ must be $C^\ell$-continuous, for what value $\ell$?

2. [Similar to Exercise VIII.6, page 211. See Figure VIII.9 also on page 211.] Consider a degree 2, order 3 B-spline curve defined with the knot vector $[0, 0, 0, 1, 2, 3, 4, 4, 4]$. How many control points does this curve need to be well-defined? Give the formulas for the following functions:
   a. The functions $N_{0,2}, N_{1,2}, N_{2,2}, N_{3,2}$. [Hint: They are piece-wise linear].
   b. Only on the interval $[0, 1]$: the functions $N_{0,3}, N_{1,3}, N_{2,3}$.
   Verify that $N_{0,3}(u) + N_{1,3}(u) + N_{2,3}(u)$ is equal to 1.

3. Page 231 expresses a system of linear equations using a tridiagonal matrix. The algorithm on that page solves that system to obtain values for the $p_i$’s from values for the $q_i$’s. A tridiagonal matrix is also called a “band matrix with $k_1 = k_2 = 1$”. The value $k_1$ is the number of non-zero entries allowed immediately before the diagonal entry, the value $k_2$ is the number of non-zero entries allowed immediately after the diagonal entry.
   This can be generalized to a “band matrix with $k_1 = 1$ and $k_2 = 2$” by allowing one more non-zero entry $\delta_i$ on each $i$-th row $(1 \leq i \leq n-1)$. Assume the first row is unchanged and still 1 0 0 0 · · ·; the last row is likewise unchanged; the $i$-th row has “$\alpha_i \beta_i \gamma_i 0$” replaced with “$\alpha_i \beta_i \gamma_i \delta_i$” and otherwise all zero. You may assume (without checking) that a divide-be-zero condition never occurs.
   Problem: Rewrite the algorithm on page 231 to work with matrices of this form.