Math 155B — Computer Graphics — Spring 2017
Homework #2 — Due 9:00pm, Friday, May 5
Hand in via GradeScope

Please show work and label answers clearly.

1. Consider a degree 3 (order 4) B-spline curve $\mathbf{q}(u)$ defined with the knot vector $[0, 0, 1, 2, 3, 3, 4, 5, 5, 5, 6, 7, 8, 9, 10]$.

a. How many control points $\mathbf{p}_0, \mathbf{p}_1, \ldots$ are needed to define the curve $\mathbf{q}(u)$?

a. What is the domain of $\mathbf{q}(u)$?

b. What is the domain of $N_{0,4}(u)$?

c. What is the domain of $N_{1,4}(u)$?

d. What is the domain of $N_{2,4}(u)$?

e. What is the domain of $N_{6,4}(u)$?

f. At $u = 2$, $\mathbf{q}(u)$ must be C^ℓ-continuous, for what value ℓ?

g. At $u = 3$, $\mathbf{q}(u)$ must be C^ℓ-continuous, for what value ℓ?

h. At $u = 5$, $\mathbf{q}(u)$ must be C^ℓ-continuous, for what value ℓ?

i. At $u = 5.5$, $\mathbf{q}(u)$ must be C^ℓ-continuous, for what value ℓ?

2. [Similar to Exercise VIII.6, page 211. See Figure VIII.9 also on page 211.] Consider a degree 2, order 3 B-spline curve defined with the knot vector $[0, 0, 0, 1, 2, 3, 4, 4, 4]$. How many control points does this curve need to be well-defined? Give the formulas for the following functions:

a. The functions $N_{0,2}$, $N_{1,2}$, $N_{2,2}$, $N_{3,2}$. [Hint: They are piece-wise linear].

b. Only on the interval $[0, 1]$: the functions $N_{0,3}$, $N_{1,3}$, $N_{2,3}$.

Verify that $N_{0,3}(u) + N_{1,3}(u) + N_{2,3}(u)$ is equal to 1.

3. Page 231 expresses a system of linear equations using a tridiagonal matrix. The algorithm on that page solves that system to obtain values for the $\mathbf{p}_i$'s from values for the $\mathbf{q}_i$'s. A tridiagonal matrix is also called a "band matrix with $k_1 = k_2 = 1$". The value k_1 is the number of non-zero entries allowed immediately before the diagonal entry, the value k_2 is the number of non-zero entries allowed immediately after the diagonal entry.

This can be generalized to a "band matrix with $k_1 = 1$ and $k_2 = 2$" by allowing one more non-zero entry δ_i on each i-th row ($1 \le i \le n-1$). Assume the first row is unchanged and still $1\ 0\ 0 \cdots$; the last row is likewise unchanged; the i-th row has "$\alpha_i\ \beta_i\ \gamma_i\ 0$" replaced with "$\alpha_i\ \beta_i\ \gamma_i\ \delta_i$" and otherwise all zero. You may assume (without checking) that a divide-be-zero condition never occurs.

Problem: Rewrite the algorithm on page 231 to work with matrices of this form.