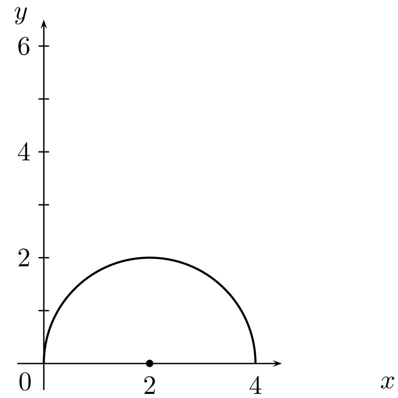
QUESTIONS FROM PRIOR 155B COURSES:

6. [20 points]

a. A circle has radius 2 and is centered at $\langle 2, 0\rangle$. Express the upper half of the circle as a degree two Bézier curve. How are the control points expressed in homogeneous coordinates? Where are the control points located in the xy-plane?



b. Use the de Casteljau algorithm to split the above curve (the upper half circle curve) into two pieces. This will express the upper half circle as a piece-wise Bézier curve: each piece should be a degree two Bézier curve.

What are the control points for these two degree two Bézier curves? Express them in homogeneous coordinates. Where are the control points located in the plane? Draw and label them on the figure.

6. In ray tracing it is often necessary to compute reflection and transmission vectors. Let $\mathbf{u}$ be the direction of a ray hitting a surface with normal $\mathbf{n}$, $\mathbf{u}$ and $\mathbf{n}$ are unit vectors.. Let the index of refraction be ν.

Give formulas for the reflection vector $\mathbf{r}$ and the transmission vector $\mathbf{t}$ (or algorithms to compute these).

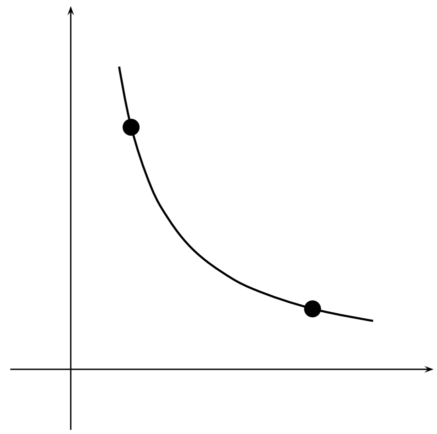
8. [20 points] Discuss different types of *distributed* ray tracing (enhancements to basic ray tracing). For full credit, list at least four different techniques used in distributed ray tracing, and explain their purposes.

1. There are two Bézier patches of degree 3 by 3. The first is defined from the control points $p_{i,j}$, for $0 \leqslant i, j \leqslant 3$. The second is defined from control points $q_{i,j}$, for $0 \leqslant i, j \leqslant 3$. The "top" of the first patch concides with the "bottom" of the second patch, i.e., $p_{i,3} = q_{i,0}$ for $i = 0, 1, 2, 3$.

a. Given a necessary and sufficient condition for the patches to form a surface that is C^1-continuous at their boundary.

b. Give a sufficient condition for the patches to form a surface that is G^1-continuous at their boundary.

3. The graph of the function $f(x) = 1/x$ is a hyperbola. Express the section of the graph between the points $\langle \frac{1}{2}, 2\rangle$ and $\langle 2, \frac{1}{2}\rangle$ as a rational Bézier curve of degree two.



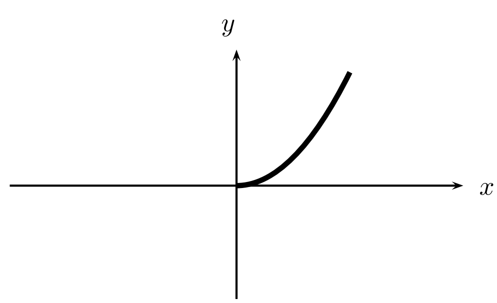
8. Consider an orientation specified with the parameters:

Yaw is 180 degrees, Pitch is 0 degrees, and Roll is 180 degrees.

a. Express this orientation as a rotation matrix.

b. Express the orientation as a unit quaternion (QUATERNIONS ARE NOT ON MIDTERM 2017).

2. Let $\mathbf{q}(u)$ be the quadratic curve defined by $\mathbf{q}(u) = \langle u, u^2\rangle$ for $0 \leqslant u \leqslant 1$. Note that $\mathbf{q}(u)$ traces out a portion of a parabola.



a. Express $\mathbf{q}(u)$ as a degree 2 Bezier curve, by giving its control points. Use a rational curve if necessary.

b. Express $\mathbf{q}(u)$ as a degree 3 Bezier curve, by giving its control points. Again, use a rational curve if necessary.

4. A B-spline curve $\mathbf{q}(u)$ of order 3 has knot vector

$$[0, 0, 0, 1, 2, 2, 3, 4, 5, 6, 7].$$

a. How many control points does this curve have?

b. For what values of i must the curve be C^i-continuous at $u = 2$?

c. For what values of i must the curve be C^i-continuous at $u = 3$?

d. For what values of i must the curve be C^i-continuous at $u = 2.5$?

e. What is the domain of the curve? (I.e., for what values of u is $\mathbf{q}(u)$ defined?)

5. The front half of a cylinder of radius 2 consists of the points

$$\{\langle x, y, z\rangle : x^2 + z^2 = 4 \text{ and } 0 \leqslant y \leqslant 1 \text{ and } z \geqslant 0\}$$

Express this surface as a single Bezier patch (of any order you wish). Define the surface by giving its control points. You should use a rational surface if necessary.

3. Recall that a single rational Bézier curve can describe a semicircle. Describe how to combine two such Bézier curves of degree three to form a single B-spline curve that traces out a complete circle of radius 1 centered at the origin. Give the knot vector and control points for your B-spline curve. Your B-spline curve should be of degree three, and be defined on the interval $[0, 2]$, and should be C^∞-continuous everywhere in $(0, 2)$ except at $u = 1$.