Math 155B — Computer Graphics — Spring 2018
Homework #1 — Due 9:00pm, Monday, April 23
Hand in via GradeScope

1. Let $\mathbf{q}(u)$ be a degree 2 Bezier curve with control points $\langle 0,0,0\rangle$, $\langle 3,6,0\rangle$, and $\langle 6,-6,0\rangle$. Express $\mathbf{q}(u)$ as a degree three Bezier curve (by giving its control points). Also express $\mathbf{q}(u)$ as a degree four Bezier curve.
2. Suppose a degree 3×2 Bezier patch with control points $\mathbf{r}_{i,j}$ is defined by

$$\mathbf{q}(u) \;=\; \sum_{i=0}^{3}\sum_{j=0}^{2} B_i^3(u)B_j^2(u)\mathbf{r}_{i,j}.$$

Express this patch as a degree 3×3 Bezier patch with control points $p_{i,j}$. Your answer should give formulas for the $\mathbf{p}_{i,j}$'s in terms of the $\mathbf{r}_{i,j}$. Give a brief proof of the correctness of your answer. (Hint: use Theorem VII.8.)
3. Express the "right half" of the unit sphere,

$$\{\langle x,y,z\rangle : x\geq 0 \text{ and } x^2+y^2+z^2=1\},$$

as a single rational Bezier patch.
4. Exercise VII.14, page 184.

5. Exercise VII.20, page 186. (You may find it easier to first think about exercise VII.19 instead.)

6. Exercise VII.24, page 191.

7. (Linear interpolation in homogeneous coordinates.) Suppose $w,v>0$. Let $\langle w\mathbf{x},w\rangle$ and $\langle v\mathbf{y},v\rangle$ be homogeneous representations of two points $\mathbf{x}$ and $\mathbf{y}$ in $\mathbb{R}^n$. For $0\leq u\leq 1$, define $\mathbf{q}(u)$ to be the point in $\mathbb{R}^n$ represented in homogeneous coordinates by the $(n+1)$-tuple

$$\textsc{Lerp}(u,\ \langle w\mathbf{x},w\rangle,\ \langle v\mathbf{y},v\rangle).$$

Evaluate the first derivative $\mathbf{q}'(u)$ of $\mathbf{q}(u)$. What are $\mathbf{q}'(0)$ and $\mathbf{q}'(1)$? Can you express your answer in terms of w, v, and $\mathbf{y}-\mathbf{x}$? Does your answer agree with the case of ordinary linear interpolation where $v=w=1$?