1. Consider the rotation matrix \[
\begin{pmatrix}
0 & 0 & 1 \\
1 & 0 & 0 \\
0 & 1 & 0
\end{pmatrix}
\]. Express this in terms of yaw, pitch and roll in two different ways. Your two answers should not just differ by multiples of 360°.

2. Express the following rotations (expressed as compositions) with yaw (y), pitch (p) and roll (r):
   a. \( R_{90°, k} \circ R_{90°, i} \circ R_{90°, j} \).
   b. \( R_{90°, j} \circ R_{90°, i} \circ R_{90°, k} \).
   c. \( R_{90°, j} \circ R_{90°, k} \circ R_{90°, i} \).

3. Let \( q_1 = (\frac{\sqrt{2}}{2}, 0, \frac{\sqrt{2}}{2}, 0) \), \( q_2 = (\frac{\sqrt{2}}{2}, 0, 0, \frac{\sqrt{2}}{2}) \), \( q_3 = (2, 0, 0, 0) \), and \( q_4 = (1, 0, 1, 0) \).
   a. Calculate \( q_1 + q_2 \), \( q_1 - q_2 \), and \( q_1 + q_3 \).
   b. Calculate the products \( q_1 q_2 \), \( q_2 q_1 \), \( q_1 q_3 \) and \( q_1(q_2 + q_3) \).
   c. Calculate \( q_1^* \), \( q_2^* \), and \( q_3^* \).
   d. Calculate \( ||q_1|| \), \( ||q_2|| \), \( ||q_3|| \), \( ||q_4|| \), \( ||q_1 q_2|| \), and \( ||q_1 q_3|| \).
   e. Calculate \( q_1^{-1} \), \( q_2^{-1} \), \( q_3^{-1} \), and \( q_4^{-1} \).

4. Let \( q_1 \) and \( q_2 \) be arbitrary quaternions. Prove that \( (q_1 q_2)^* = q_2^* q_1^* \) and that \( (q_1 q_2)^{-1} = q_2^{-1} q_1^{-1} \).

5. Let \( q_1 \) and \( q_2 \) be arbitrary quaternions. Prove that \( ||q_1 q_2|| = ||q_1|| \cdot ||q_2|| \). [Hint: You can give a short proof using the previous exercise and the equality \( q q^* = q^* q = ||q||^2 \).]

6. Express the matrix from Problem 1 as a unit quaternion in two different ways.