1. Consider the knot vector \([0, 0, 0, 0, 1, 3, 3, 4, 5, 5, 5, 6, 7, 8, 8, 8, 8]\) with 17 knots \(u_0, \ldots, u_{16}\). E.g., \(u_4 = 1\) and \(u_7 = 4\) and \(u_{11} = 6\). Let this define a degree three B-spline curve \(q(u)\) with \(n + 1\) control points \(p_0, \ldots, p_n\). Answer questions a.-i. below. You will need to make a copy of this graph:

\[
\begin{array}{c}
\text{y} \\
\hline
0 & 1 & 3 & 4 & 5 & 6 & 7 & 8 \\
\hline
x
\end{array}
\]

a. How many control points are needed for this curve? What is the value of \(n\)?
b. What is the largest value \(i\) such that \(N_{i,4}(u)\) is defined?
c. What is the domain of the B-spline curve \(q(u)\)?
d. Make a copy of the graph shown above. Draw and label the graph of \(N_{0,4}\) on the graph.
e. What is the support of \(N_{0,4}\)?
f. What is the support of \(N_{4,4}\)?
g. Draw and label the graph of \(N_{5,4}\). You can do this on the same graph as used in part d.
h. For what value \(\ell\) (if any) is it guaranteed that \(q(u)\) is \(C^\ell\)-continuous at \(u = 3\)?
i. For what pairs of integers \(i\) and \(j\) must it be that \(q(i) = p_j\)? (Explicitly list all of them.)

2. Consider the knot vector \([0, 0, 0, 0, 1, 1, 1, 2, 2, 3, 3, 3, 3]\) for a degree three B-spline curve \(q\).

a. The curve \(q\) uses control points \(p_0, p_1, \ldots, p_\ell\). How many control points does this curve use? What is \(\ell\) equal to?
b. How many blending functions \(N_{i,4}(u)\) does this curve use?
c. What is the support of \(N_{3,4}(u)\)? What is the support of \(N_{4,4}\)? For full credit, correctly indicate whether the endpoints of the intervals of support are included in the supports.
d. The B-spline curve $q$ can be written as a union of Bézier curves of degree 3. How many Bézier curves are used, and what are their control points?

e. Must the B-spline curve $q$ be continuous on the domain $[0, 3]$? Must $q$ be $C^1$-continuous on the domain $[0, 3]$?

f. The derivative $q'$ can be expressed as a B-spline curve of degree two. What is the knot vector for $q'$? Will the derivative $q'(u)$ be well-defined and continuous for all $u$ in the interval $(0, 3)$? If not, at what values $u \in (0, 3)$ might it not be well-defined and continuous?

3. Page 315 in the textbook (version B.i.k) shows three equations labelled (IX.7) for the blending functions $N_{1,3}(u)$, $N_{2,3}(u)$ and $N_{3,3}(u)$ for the knot vector $0, 0, 0, 0, 1, 1, 1, 1$. Carry out and show the work need to derive these three equations. [You may start with the equations for $N_{2,2}(u)$ and $N_{3,2}(u)$ as given in the text; all of the other functions $N_{i,2}(u)$ (for $i = 0, 1, 4, 5$) are equal to the constant zero function.]

4. Give a full acknowledgement of assistance. This includes anyone, any written source, any web site, etc., that helped you; and anyone you helped.