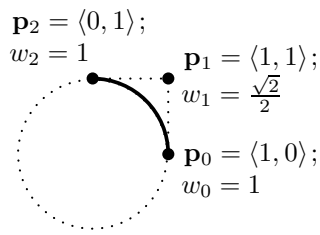
Math 155A - Computer Graphics

Homework Assignment, due April 18, 2001

1. Let $\mathbf{q}(u)$ be the rational, degree 2 Bézier curve with homogeneous control points $\mathbf{p}_0 = (1, 0, 1)$, $\mathbf{p}_1 = (1/\sqrt{2}, 1/\sqrt{2}, 1/\sqrt{2})$ and $\mathbf{p}_2 = (0, 1, 1)$. Prove that this Bézier curve traces out the $90°$ arc of the unit circle in $\mathbb{R}^2$ from the point $(1, 0)$ to $(0, 1)$.



2. This problem concerns the "standard" quadratic B-spline knot vector, see Hill, pp. 633-635.

a. Derive the formulas for the (a) order 0, (b) order 1, and (c) order 2 blending functions, $N_{i,m}$, for $m = 1, 2, 3$ and all appropriate values for i; which are based on the knot vector $[0, 0, 0, 1, 2, 3, 3, 3]$.

b. Graph the functions (use separate graphs for order 1, order 2 and order 3).

c. How many control points are needed to define a quadratic (degree 2, order 3) B-spline curve with these control points.

d. What is the domain of such a B-spine curve?

e. In terms of the control points, $\mathbf{p}_i$, what are the starting and ending points of the B-spline curve?