Name: ____________________________ ID#: ____________________

Math 15A - Discrete Mathematics - Spring 1999

Midterm Review Problems — May 3

On the midterm, you may use a *single* sheet of notes (double-sided, $8\frac{1}{2} \times 11$ paper). You may not use the textbook or a calculator. This sheet may not be shared with other students: you must have your own separate sheet of notes.

The midterm will cover all the course material to-date, through section 4.3.

1. (a) Consider the sequence $1, -2, 4, -8, 16, -32, \ldots$. Letting $a_0 = 1$, $a_1 = -2$, etc., give a general formula for a_i.

   (b) Give a formula for $\sum_{i=0}^{n} a_i$.

2. (a) Express the integer 85 in binary (base 2) notation.

   (b) Express the negative integer -85 in binary notation (using a 8-bit two's complement representation).

3. Convert the propositional formula

   $$p \to (q \to (r \to (s \to t)))$$

   into an equivalent formula using only the connectives $\wedge$, $\vee$ and $\sim$.

4. Express the formula $(p \leftrightarrow q) \leftrightarrow r$ as a Boolean circuit using AND, OR and NOT gates.

5. Let $L(x, y)$ be the predicate "x loves y", and let $H(x)$ be the predicate "x is handsome". Let quantified variables in $(\forall x)$ and $(\exists y)$ range over all people. Express the following formally using quantifiers, Boolean connectives and the two predicates:

   (a) Someone loves everyone.

   (b) Everyone is loved by someone.

   (c) Some people love no one.

   (d) Any handsome person is loved by everyone.

   (e) There is a handsome person who is loved by no one.

   (f) For a person to be loved by everyone it is necessary that they be handsome.

6. Which of the following are tautologies?

   (a) $(\sim(p \wedge \sim q)) \leftrightarrow (\sim p \vee q)$

   (b) $p \to ((p \to q) \to p)$

   (c) $(p \leftrightarrow (q \leftrightarrow r)) \leftrightarrow ((p \leftrightarrow q) \leftrightarrow r)$.

7. Express the following assertions (semi-)formally. You may use quantifers such as "$\forall$ rational x", "$\exists$ positive integer y", Boolean connections, and predicates such as "$x < y$", "$x \leq y$", "$x = y$" "$x \neq y$", and functions addition, subtraction, multiplication, division and square roots.

   (a) There is a smallest positive integer.

   (b) There is no smallest positive rational number.

   (c) For any irrational $x < y$, there is a rational z between x and y.

   (d) For an integer n to be prime, it is sufficient that n has at most one divisor which is greater than or equal to $\sqrt{n}$.

8. Which of the assertions (a)-(d) of the previous problem are true?

9. Prove or disprove the following assertion:

   Assertion: Let n be an odd integer. Then n^3 mod 8 is equal to n mod 8.

10. Prove or disprove the following assertion: (You may assume without proof that $\sqrt{2}$ is irrational if you need to. Any other needed irrational number should be proved to be irrational.)

    Assertion: For all irrational numbers a, b with $a + b \neq 0$, the number $a/(a + b)$ is irrational.

11. Prove the following by induction.

    Theorem: $\sum_{n=0}^{k-1} 3n(n+1) = k^3 - k$.