

Name: _____

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Math 15A - Discrete Mathematics - Spring 1999

Quiz #7 — May 26

You may NOT use the textbook, notes or other references for this test.

1. (40 pts) For each of the following functions f , state whether f is (a) one-to-one and (b) onto. If it is not one-to-one, give an example of two domain elements where f has the same value. Also, if it is not onto, give an example of an element in the codomain of f , which is not in the range of f . (Watch the domains and codomains carefully!)

a. $f : \mathbb{R} \rightarrow \mathbb{R}, f(x) = 2x$.

ANSWER: This is one-to-one and onto.

b. $f : \mathbb{R} \rightarrow \mathbb{R}, f(x) = x^2$.

ANSWER: This is neither one-to-one and onto.

No negative number is in the range of f .

Also: $f(1) = f(-1)$ (for instance).

c. $f : \mathbb{N} \rightarrow \mathbb{N}, f(x) = 2x$.

ANSWER: This is one-to-one but not onto.

No odd integer is in the range of f .

- d. Let $g : \mathbb{R}^+ \rightarrow \mathbb{R}$ and $h : \mathbb{R} \rightarrow \mathbb{R}$ be defined by $g(x) = \log_e x$ and $h(x) = e^x$. Then define $f : \mathbb{R}^+ \rightarrow \mathbb{R}$ to be $f = h \circ g$.

ANSWER: f is one-to-one but not onto.

The range of f is only \mathbb{R}^+ .

- e. Let $g : \mathbb{R}^+ \rightarrow \mathbb{R}$ by $g(x) = \log_e x$ and let $f = g^{-1}$.

ANSWER: $f : \mathbb{R} \rightarrow \mathbb{R}^+$ is one-to-one and onto.

2. (20 pts) (a) Let A be a set with n elements and B a set with m elements, with $n \leq m$. How many different one-to-one functions are there with domain A and codomain B ?

ANSWER: $\frac{m!}{(m-n)!}$

- (b) Now suppose A has n elements and B has m elements and that $n > m$. How many different one-to-one functions are there with domain A and codomain B ?

ANSWER: Zero. There are no such functions.