Name: ______________________________ ID#:____________________

Math 15A - Discrete Mathematics - Spring 1999

Quiz #8 ANSWER KEY — June 2

You may NOT use the textbook, notes or other references for this test. There are four problems, with the test continueing on the reverse. For this quiz: all relations are presumed to be binary relations.

1. (10 pts) **(a)** Name the three conditions that R must satisfy in order to be an equivalence relation? (It is enough to just give the names, you do not have to write out their definitions.)

   ANS: reflexive, symmetric and transitive.

   (b) Name the three conditions that R must satisfy in order to be a partial order.

   ANS: reflexive, antisymmetric and transitive.

2. (16 pts) Let R be the relation on $\mathbb{Z}^+$ defined by

$$nRm \quad \text{if and only if} \quad \gcd(n,m) > 1.$$

   That is, nRm holds iff n and m have a prime factor in common. Does R have the following properties? If you answer "No" to a property, give an example to show why the answer is No.

   a. Is R reflexive? YES

   b. Is R symmetric? YES

   c. Is R antisymmetric? NO. $2R6$ and $6R2$ but $2 \neq 6$.

   d. Is R transitive? NO. $2R6$ and $6R3$ but not $2R3$.

3. (20 pts) Let S be the relation on $\mathbb{Z}^+$ defined by

$$nSm \quad \text{if and only if} \quad (\forall \text{ primes } p, \text{ if } p|n \text{ then } p|m)$$

Does S have the following properties? If you answer "No" to a property, give an example to show why the answer is No.

a. Is S reflexive? YES

b. Is S symmetric? NO. $2S6$ but not $6R2$.

c. Is S antisymmetric? NO. $2S4$ and $4S2$, but $2 \neq 4$.

d. Is S transitive? YES.

e. Is S a total order? NO. IT IS NOT ANTISYMMETRIC. ALTERNATIVELY, SINCE NEITHER $2S3$ NOR $3S2$ HOLD.

4. (14 pts) Draw the Hasse diagram of the $\subseteq$ relation on $\wp(\{a, b, c\})$.

THE HASSE DIAGRAM HAS 8 elements. The highest one is $\{a, b, c\}$. It has undirected edges down to the three elements $\{a, b\}$, $\{b, c\}$ and $\{a, c\}$. Below these are the three elements $\{a\}$, $\{b\}$ and $\{c\}$, with edges from $\{a\}$ up to $\{a, b\}$ and $\{a, c\}$, and edges from $\{b\}$ up to $\{a, b\}$ and $\{b, c\}$ and edges from $\{c\}$ up to $\{a, c\}$ and $\{b, c\}$. The lowest element is $\emptyset$ and from it there are edges up to $\{a\}$, to $\{b\}$ and to $\{c\}$.