1. (Distributivity or non-distributivity for $\land$ over $\iff$.) For each of the following either prove it is true, or give a truth assignment $\varphi$ which shows it is false. (To give a proof that it is true, you may either argue informally, or use truth tables or reduced truth tables.)

(a) $p \land (q \iff r) \vdash (p \land q) \iff (p \land r)$.

(b) $(p \land q) \iff (p \land r) \vdash p \land (q \iff r)$.

2. (Semantics of $\iff$.)

(a) Show that $\iff$ is associative by proving that $p \iff (q \iff r) \models (p \iff q) \iff r$.

(b) Consider a formula $p_1 \iff p_2 \iff p_3 \iff \ldots \iff p_k$.

Give a simple characterization of when $\varphi(p_1 \iff p_2 \iff p_3 \iff \ldots \iff p_k) = T$. Your answer should be in terms of the number of $i$'s such that $\varphi(p_i)$ is equal to $T$ or $F$.

3. (Equivalence to a single formula.) Let $\Gamma$ be a set of formulas.

(a) Suppose $\Gamma$ is finite. Prove that there exists a formula $A$ such that $\Gamma \models A$ and $A \models \Gamma$.

(The latter means that for all $B \in \Gamma$, $A \models B$.)

(b) Give an example of an infinite, satisfiable $\Gamma$ for which there does not exist a satisfiable formula $A$ such that $A \models \Gamma$. 