1. Suppose that $\Gamma \models p_i$ or $\Gamma \models \neg p_i$, for every $i$. Prove that, for every formula $A$, $\Gamma \models A$ or $\Gamma \models \neg A$. (This property of $\Gamma$ is similar to being complete; however, instead of having one of $A$ or $\neg A$ a member of $\Gamma$, we have one of $A$ or $\neg A$ tautologically implied by $\Gamma$.)

2. Use the Compactness Theorem for propositional logic to prove that a graph is 3-colorable if and only if every finite subgraph is 3-colorable. (“3-colorable” means there is an assignment of three colors to the vertices of the graph so that no edge connects vertices assigned the same color.) For this, fix a graph $G$. Use propositional variables $r_i$, $g_i$, $b_i$ whose intended meanings are that “Vertex $i$ is red”, “Vertex $i$ is green”, and “Vertex $i$ is blue”, respectively. Let $\Gamma$ be a set of formulas using these variables that expresses the conditions that (a) each vertex has a color assigned to it, and (b) if two vertices $i$ and $j$ are joined by an edge in $G$, then they are not assigned the same color. The set $\Gamma$ should be satisfiable if and only if $G$ is 3-colorable. Then apply the Compactness Theorem.

This is mostly a conceptual problem. Feel free to discuss this on piazza and discord. What to hand-in to be graded: Describe what formulas are in the set $\Gamma$ in terms of the vertices and edges of $G$. 