\[ \text{TFA and } \Pi \vdash \eta \Rightarrow \Pi \vdash A \]

\[ \text{TFA and } \Pi \vdash \eta \Rightarrow \Pi \vdash A \]

\[ \{ B : \Pi + B \} \cup \{ B : \Pi \vdash B \} \]

Consequences of \( \Pi \); Theorems of \( \Pi \).

\( \Pi \) is a theory if \( \Pi = \{ \exists A : \Pi \vdash A \} \)

\( T \) \text{ is } \eta \vdash \eta \]

\( \neg \exists A + A \Rightarrow \exists \exists A, \neg A \) is inconsistent
Yes it is adequate.
\( \{ \land, \oplus \} \) is not adequate

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<th>( p )</th>
<th>( \neg p )</th>
<th>( p \oplus p )</th>
<th>( \neg (p \oplus p) )</th>
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Inconclusive

Question: Can we express \( \land \lor \neg \) or \( \land \rightarrow \) with \( \oplus \)?

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<th>( p \oplus q )</th>
<th>( \neg (p \oplus q) )</th>
<th>( \neg p )</th>
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\( p \oplus q \neq 1 = q \oplus p \)

\( p \oplus (q \oplus p) \neq 1 = (p \oplus q) \lor r \)

\( p \oplus (q \oplus r) = 1 \neq (p \oplus q) \lor (q \oplus r) \)

\( p \equiv q \lor r \equiv 2 \)
\( T \cup (A \lor B) \) is inconsistent \( \iff \ T \cup (A \land B) \) and \( T \cup (A \land B) \) are inconsistent.

\( \implies \) (Easy direction)
\begin{align*}
&\text{Know } A \land A \lor B, \quad \text{and } A \lor A \land B \\
&\text{Know } B \lor A \land B, \quad \text{and } B \lor B \land A
\end{align*}
\( B \lor A \land B \) is a PL-axiom
\( B \lor B \land A \implies B \land \neg A \implies B \)

Let \( C \) be any formula.
Want to show \( T, A \lor C, \) and \( T, B \land C \)

By \( A \lor A \land B, \) \( B \lor A \land B, \) and by the assumption that \( T, A \lor B \lor C \) (\( T \cup A \lor B \) is inconsistent), we get \( T, A \lor C \) and \( T, B \land C \).

\( T \cup A \lor A \land B \) and \( T, A \lor B \lor C \)

By Hypothetical Syllogism, \( T \cup A \implies C \)
\( \therefore T, A \lor C \).
Assume $T, A$ in consistent and $T, B$ in consistent.

Want to show $T \cup \{A \lor B\}$ is inconsistent.

Let $C$ be arbitrary. Want to $T, A \lor B \vdash C$

Use proof-by-cases:

Show $T, A \lor B, A \vdash C$

and Show $T, A \lor B, \neg A \vdash C$

i.e. $T, \neg A \rightarrow B, \neg A \vdash C$

$T, \neg A \rightarrow B, \neg A \vdash B$ by M.P.

and $T \vdash B \rightarrow C$ since $T \cup \{B\}$ is inconsistent.

So $T, \neg A \rightarrow B, \neg A \vdash C$ MP.

$T, A \vdash C$ since $T \cup \{A\}$ is inconsistent.

Thus $T, A \lor B, A \vdash C$.
Proof by cases

\[ T + A \iff T, B \vdash A \text{ and } T, \neg B \vdash A \]

i.e. \[ T + A \iff T \vdash B \rightarrow A \text{ and } T \vdash \neg B \rightarrow A \]

Two cases \( B \) and \( \neg B \)

Generalized Proof by Cases

Suppose \( T \vdash D \lor E \)

Then \( T + A \iff T \vdash D \rightarrow A \text{ and } T \vdash E \rightarrow A \)

Proof omitted