For all the problems, let $\Gamma$ be the set of formulas
\[ \{p_{i+1} \rightarrow p_i : i \geq 1\} \cup \{p_j \rightarrow p_k : j \text{ is prime, and } k \text{ is the least prime } > j\}. \]
In other words, $\Gamma$ is:
\[ \{p_2 \rightarrow p_1, p_3 \rightarrow p_2, p_4 \rightarrow p_3, \ldots\} \cup \{p_2 \rightarrow p_3, p_3 \rightarrow p_5, p_5 \rightarrow p_7, p_7 \rightarrow p_{11}, \ldots\}. \]

1. Is $\Gamma$ satisfiable? If so, describe all the truth assignments that satisfy $\Gamma$.
   - Yes, $\Gamma$ is satisfiable. It has three satisfying assignments.
   - $\phi(p_i) = T$ for all $i$.
   - $\phi(p_i) = F$ for all $i$.
   - $\phi(p_i) = T$ and $\phi(p_i) = F$ for all $i > 2$.

2. Does $\Gamma \models p_1$?
   If so, give the minimal subset $\Gamma_0$ of $\Gamma$ such that $\Gamma_0 \models p_1$.
   - No. (Because the second satisfying assignment above has $\phi(p_1) = F$.)

3. Does $\Gamma \models p_1 \rightarrow p_1$?
   If so, give the minimal subset $\Gamma_1$ of $\Gamma$ such that $\Gamma_1 \models p_1 \rightarrow p_1$.
   - Yes. $p_1 \rightarrow p_1$ is a tautology, so $\not\exists \phi(p_1) \neq T$.
   - $\Gamma_1 = \emptyset$ (the empty set).

4. Does $\Gamma \models p_2 \rightarrow p_8$?
   If so, give the minimal subset $\Gamma_2$ of $\Gamma$ such that $\Gamma_2 \models p_2 \rightarrow p_8$.
   - Yes.
   - $\Gamma_2 = \{p_2 \rightarrow p_3, p_3 \rightarrow p_5, p_5 \rightarrow p_7, p_7 \rightarrow p_{11}, p_{11} \rightarrow p_{10}, p_{10} \rightarrow p_9, p_9 \rightarrow p_8\}$. 