

Let  $f(x) = 2x^3 - 9x^2 + 12x + 1$ , on the domain  $[0, 3]$ .

1. What is  $f'(x)$ ?
2. What are the critical points of  $f$ ?
3. Does  $f$  have a maximum value on  $[0, 3]$ ; if so, what is it?
4. Does  $f$  have a minimum value on  $[0, 3]$ ; if so, what is it?
5. List all local minima and local maxima of  $f$  in  $(0, 3)$ .

$$\textcircled{1} \quad f'(x) = 6x^2 - 18x + 12 = 6(x^2 - 3x + 2) = 6(x-1)(x-2)$$

$$\textcircled{2} \quad f'(x) \text{ is defined for all } x.$$

$$f'(x) = 0 \text{ for } x=1 \text{ and } x=2.$$

The critical points are at  $x=1$  and  $x=2$ .

$$\textcircled{3}-\textcircled{5} \quad f(0) = 1$$

$$f(1) = 2 - 9 + 12 + 1 = 6$$

$$f(2) = 2 \cdot 8 - 9 \cdot 4 + 12 \cdot 2 + 1 = 16 - 36 + 24 + 1 = 5$$

$$f(3) = 2 \cdot 27 - 9 \cdot 9 + 12 \cdot 3 + 1 = 54 - 81 + 36 + 1 = 10$$

$$\textcircled{3} \quad \text{Maximum value of } f \text{ on } [0, 3] \text{ is } 10$$

$$\textcircled{4} \quad \text{Minimum value of } f \text{ on } [0, 3] \text{ is } 1$$

$$\textcircled{5} \quad f(x) \text{ has a local minimum value of } 5 \text{ at } x=2$$

$$f(x) \text{ has a local maximum value of } 6 \text{ at } x=1.$$

Alternate method to identify local minimum + maximum:

$$f'(x) > 0 \text{ for } x < 1$$

$$f'(x) < 0 \text{ for } 1 < x < 2$$

$$f'(x) > 0 \text{ for } x > 2$$

$$\left. \begin{array}{l} f'(x) > 0 \text{ for } x < 1 \\ f'(x) < 0 \text{ for } 1 < x < 2 \end{array} \right\} \Rightarrow f \text{ has a local maximum at } x=1$$

$$\left. \begin{array}{l} f'(x) < 0 \text{ for } 1 < x < 2 \\ f'(x) > 0 \text{ for } x > 2 \end{array} \right\} \Rightarrow f \text{ has a local minimum at } x=2.$$