1. State the definition of “the rank of $A$”. **ANSWER:** The dimension of the row space of $A$. (It also equals the dimension of the column space of $A$.)

2. Now let $A$ be the matrix $A = \begin{pmatrix} 1 & 2 & 1 & 1 & 1 \\ 1 & 2 & 2 & 3 & 2 \\ 1 & 2 & 2 & 3 & 3 \\ 1 & 2 & 2 & 3 & 4 \end{pmatrix}$. Answer questions a.-e.

   a. Calculate the rank $A$.
   b. Calculate the nullity $A$.
   c. Give a basis for the column space of $A$.
   d. Give a basis for the row space of $A$.
   e. Give a basis for the null space of $A$.

   **ANSWER:** Use row operations to find the row echelon form of $A$ as

   \[
   \begin{pmatrix} 1 & 2 & 1 & 1 & 1 \\ 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}.
   \]

   Since $A$’s ref form has three lead variables, $A$ has rank 3. Since there are 2 free variables, $A$ has nullity 2.

   The lead variables occur in columns 1, 3 and 5, so a basis for the column space of $A$ is

   \[(1, 1, 1, 1, 1)^T, (1, 2, 2, 2)^T, (1, 2, 3, 4)^T.\]

   A basis for the row space of $A$ is

   \[(1, 2, 1, 1, 1), (0, 0, 1, 2, 1), (0, 0, 0, 0, 1).\]

   Solving $Ax = 0$ using back-substitution (with the ref form of $A$) shows that a basis for the null space is:

   \[(1, 0, -2, 1, 0)^T, (-2, 1, 0, 0, 0)^T.\]