Name: Thursday section time:
Student ID:

Math 20F - Linear Algebra - Spring 2003

Quiz #4 — May 8

(Do not discuss the quiz with students who haven't taken it yet – until 8:00pm.)
You must show your work in order to get credit for a problem. **Label your answers clearly.**

1. State the definition of "the rank of A". *ANSWER: The dimension of the row space of A. (It also equals the dimension of the column space of A.)*

2. Now let A be the matrix $A = \begin{pmatrix} 1 & 2 & 1 & 1 & 1 \\ 1 & 2 & 2 & 3 & 2 \\ 1 & 2 & 2 & 3 & 3 \\ 1 & 2 & 2 & 3 & 4 \end{pmatrix}$. Answer questions a.-e.

- **a.** Calculate the rank A.
- **b.** Calculate the nullity A.
- **c.** Give a basis for the column space of A.
- **d.** Give a basis for the row space of A.
- **e.** Give a basis for the null space of A.

ANSWER: Use row operations to find the row echelon form of A as

$$\begin{pmatrix} 1 & 2 & 1 & 1 & 1 \\ 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Since A's ref form has three lead variables, A has rank 3. Since there are 2 free variables, A has nullity 2.

The lead variables occur in columns 1, 3 and 5, so a basis for the column space of A is

$$(1, 1, 1, 1)^T, (1, 2, 2, 2)^T, (1, 2, 3, 4)^T.$$

A basis for the row space of A is

$$(1, 2, 1, 1, 1), (0, 0, 1, 2, 1), (0, 0, 0, 0, 1).$$

Solving $A\mathbf{x} = \mathbf{0}$ using back-substitution (with the ref form of A) shows that a basis for the null space is:

$$(1, 0, -2, 1, 0)^T, (-2, 1, 0, 0, 0)^T.$$