

Name:  
Student ID:

Thursday section time:

### Math 20F - Linear Algebra - Spring 2003

#### Answers to Self-assessment Quiz #5.5 — May 22

You must show your work in order to get credit for a problem. **Label your answers clearly.**

1. Let  $\mathbf{u}_1 = (1, 1, 1, 0)^T$  and  $\mathbf{u}_2 = (0, 1, 1, 1)^T$ . Let  $U = \text{Span}(\mathbf{u}_1, \mathbf{u}_2)$ . Find a basis for  $U^\perp$ .

ANSWER: Form the matrix  $A = \begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{pmatrix}$ . The nullspace of the matrix is equal to  $U^\perp$ , since  $R(A^T) = U$ . After finding also solutions to  $A\mathbf{x} = \mathbf{0}$  (details of work omitted), we have

$$U^\perp = \text{Span}((1, -1, 0, 1)^T, (0, 1, -1, 0)^T).$$

- 2 Find the linear function that best approximates data

$$\begin{array}{c|c|c|c|c} x & -2 & 0 & 1 & 2 \\ \hline y & 2 & 0 & 1 & 2 \end{array}$$

in the least squares sense.

ANSWER: Form the matrix  $A = \begin{pmatrix} 1 & -2 \\ 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{pmatrix}$ . Solve  $A\mathbf{x} = \mathbf{b}$ , where

$\mathbf{b} = \begin{pmatrix} 2 \\ 0 \\ 1 \\ 2 \end{pmatrix}$ . For this, compute the normal equation  $(A^T A)\mathbf{x} = A^T \mathbf{b}$ , which is

$$\begin{pmatrix} 4 & 1 \\ 1 & 9 \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}.$$

This system has the solution  $x_1 = 44/35$  and  $x_2 = -1/35$ . Thus, the answer is  $(-1/35)x + 44/35$  — this is the linear function that best approximates the data in the least squares sense.