1. Prove that a set \( S \) (of integers) is r.e. (recursively enumerable) iff is the domain of a recursive function. (For this and the next problem is it OK to work Turing machines and/or with intuitive notions of computation, i.e., to invoke the Church-Turing thesis.)

2. Prove that a set \( S \) (of integers) is r.e. iff is the range of a recursive function.

3. Prove that \( \mathbb{Q} \) does not prove \( \forall x (0 + x - x) \).

Problems 4.-11.: Suggest to do only one, and post to piazza for others to verify. Hints are given on pp. 85-86 of the *Handbook of Proof Theory*.

4. Show that \( I\Delta_0 \) proves multiplication is commutative.

5. Show that \( I\Delta_0 \) proves multiplication is associative.

6. Show that \( I\Delta_0 \) proves the distributive law for \( + \) and \( \cdot \).

7. Show that \( I\Delta_0 \) proves the cancellation law for addition:
   \[ \forall x, y, z (x + z = y + z \rightarrow x = y). \]

8. Show that \( I\Delta_0 \) proves transitivity of \( \leq \).

9. Show that \( I\Delta_0 \) proves \( \forall x, y (x + y = 0 \rightarrow x = 0 \lor y = 0) \) and
   \[ \forall x, y (x \cdot y = 0 \rightarrow x = 0 \lor y = 0). \] (Anti-idempotency)

10. Show that \( I\Delta_0 \) proves reflexivity, tricotomy, and antisymmetry of \( \leq \).

11. Show that \( I\Delta_0 \) proves the cancellation law for multiplication:
    \[ \forall x, y, z (x \cdot S z = y \cdot S z \rightarrow x = y). \]