

Math260 - Introduction to Mathematical Logic

Fall 2007 – Winter 2008

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Homework #1. Due Tuesday, October 16, 2007.

General instructions for all homeworks: Proofs are not required unless the problem asks for a proof. Don't forget conventions on omitting parentheses.

1. Simplify each of the following formulas, by giving a simpler formula that is tautologically equivalent.
 - a. $p \rightarrow \neg p$.
 - b. $(p \rightarrow q) \wedge p$.
 - c. $p \rightarrow p \wedge q$.
 - d. $(p \rightarrow q) \wedge \neg p$.
 - e. $(p \wedge q) \vee p$.
 - f. $(p \vee q) \wedge p$.
 - g. $(p \rightarrow q) \rightarrow q$.
 - h. $(p \rightarrow q) \rightarrow p$.
2. Prove that $\{\rightarrow\}$ is not a functionally complete set of connectives.
3. Let the Shaeffer stroke (NAND) be defined so that $p|q$ means the same as $\neg(p \wedge q)$. Prove that $\{| \}$ is functionally complete.
4. Let \downarrow be the NOR connective; $(p \downarrow q) \models \neg(p \vee q)$. Both $\{| \}$ and $\{\downarrow\}$ are both functionally complete. Prove that there is no other binary Boolean connective \circ such that $\{\circ\}$ is functionally complete. [Hint: there are 16 binary Boolean connectives, including constant functions and functions that depend on only one input.]
5. Use a truth table to prove that $(p \leftrightarrow q) \leftrightarrow r$ is tautologically equivalent to $p \leftrightarrow (q \leftrightarrow r)$.

6. Describe succinctly the conditions under which the formula

$$x_1 \leftrightarrow x_2 \leftrightarrow x_3 \leftrightarrow \dots \leftrightarrow x_n$$

is true. That is, give a natural characterization of when a truth assignment assigns this formula the value True.

7. Let $p_{i,j}$ be distinct variables, for $1 \leq i \leq m$ and $1 \leq j \leq n$. Consider the CNF formula

$$\varphi := \bigwedge_{i=1}^m \bigvee_{j=1}^n p_{i,j}.$$

Describe the formula φ^{DNF} that is (a) equivalent to φ and (b) is in disjunctive normal form. (Say, as obtained from φ by using distributivity of \wedge and \vee .) How large is the formula φ^{DNF} ? (Suggestion: measure the size of φ^{DNF} by the number of occurrences of variables in the formula.)