Math260 - Introduction to Mathematical Logic

Fall 2007 – Winter 2008

Instructor: Sam Buss

Homework #2. Due Thursday, October 18, 2007.

1. Show that the following are derived rules for natural deduction:

$$\frac{\neg\varphi}{\neg(\varphi\land\psi)} \qquad \frac{\neg\psi}{\neg(\varphi\land\psi)} \qquad \frac{\neg\varphi \qquad \neg\psi}{\neg(\varphi\lor\psi)}$$

2. Same as problem #1. But skip any that were done during class lecture.

$$\frac{\psi}{\varphi\rightarrow\psi} \qquad \frac{\neg\varphi}{\varphi\rightarrow\psi} \qquad \frac{\varphi \qquad \neg\psi}{\neg(\varphi\rightarrow\psi)}$$

3. Give natural deduction proofs of the following formulas.

a. $\varphi\land\neg\varphi\rightarrow\bot$.

b. $\neg(\neg p\land\neg q)\rightarrow p\lor q$.

c. $p\lor q\rightarrow\neg(\neg p\land\neg q)$.

d. $(\neg q\rightarrow\neg p)\rightarrow(p\rightarrow q)$.

4. Prove the following by giving explicit natural deduction proofs.

a. $p\land(q\lor r)\vdash(p\land q)\lor(p\land r)$.

b. $(p\land q)\lor(p\land r)\vdash p\land(q\lor r)$.

c. $p\lor(q\lor r)\vdash(p\lor q)\lor r$.

5. Let ν be a truth assignment. Let φ be a formula containing only the variables $p_1,\ldots,p_n$. For a variable p_i, let p^i_ν be p_i if $\nu(p_i)=T$ and be $\neg p_i$ if $\nu(p_i)=F$. Define φ^ν similarly to be either φ or $\neg\varphi$. Prove that $p_1^\nu,\ldots,p_n^\nu\vdash\varphi^\nu$. [Hint: Use induction on the complexity of φ, and properties like the ones in problems #1 and #2.]

6. (Alternate proof of the finite version of the completeness theorem.) Suppose $\vDash\varphi$. Use the following suggestion to prove $\vdash\varphi$.

Suggestion: Using the property in problem #5 as the base case, prove that $p_1^\nu,\ldots p_k^\nu\vdash\varphi^\nu$ using induction on k, with k varying from n down to 0. This is somewhat like exhaustively searching a truth table for φ.