Math260 - Introduction to Mathematical Logic

Fall 2007 – Winter 2008

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Homework #6. Due Thursday, February 4, 2008.

1. Let the binary representation of a be $a_m \dots a_1 a_0$. Prove that the function

$$\text{BinaryRep}(m) \;=\; \langle a_m, \dots, a_0 \rangle$$

is primitive recursive.

2. Define FromBinaryRep so that for all a and all $b = \langle \dots, 2, a_m, \dots, a_0 \rangle$, we have $\text{FromBinaryRep}(b) = a$. Prove that FromBinaryRep is primitive recursive.

3. Prove that a set X is recursively enumerable if and only if X is the domain of a partial recursive function.

4. Prove that a set X is recursively enumerable if and only if X is the range of a partial recursive function.

5. Let $H_2 = \{e : \{e\}(0)\downarrow\}$. Prove that H_2 is not recursive. [As discussed in class, this is an example of a set that that is recursively enumerable, but not recursive.]