1. Give an example of a consistent theory $T \supseteq I\Sigma_1$ such that $T$ proves $\neg \text{Con}_T$. [To be done in class.]

2. Let $A = \{ \varphi : I\Sigma_1 \vdash \varphi \}$ and $B = \{ \varphi : I\Sigma_1 \vdash \neg \varphi \}$. Prove that $A$ and $B$ are recursively inseparable, i.e., that there is no recursive set $C$ such that $A \subseteq C$ and $B \subseteq (\mathbb{N} \setminus C)$.

3. Suppose that $\mathcal{F} = (S, R)$ is a Kripke frame such that $R$ is transitive and such that there is no infinite descending $R$-chain (i.e., no sequence $s_0, s_1, s_2, \ldots$ such that $s_i Rs_{i+1}$ for all $i$). Prove that

$$\Box(\Box \varphi \rightarrow \varphi) \rightarrow \Box \varphi$$

is valid in $\mathcal{F}$ for all formulas $\varphi$. 
