Math260 - Introduction to Mathematical Logic

Fall 2007 – Winter 2008

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Homework #9. Due Tuesday, March 18, 2008.

1. Give an example of a consistent theory $T \supseteq I\Sigma_1$ such that T proves $\neg Con_T$. [To be done in class.]

2. Let $A = \{\varphi : I\Sigma_1 \vdash \varphi\}$ and $B = \{\varphi : I\Sigma_1 \vdash \neg\varphi\}$. Prove that A and B are recursively inseparable, i.e., that there is no recursive set C such that $A \subseteq C$ and $B \subseteq (\mathbb{N} \setminus C)$.

3. Suppose that $\mathcal{F} = (S, R)$ is a Kripke frame such that R is transitive and such that there is no infinite descending R-chain (i.e., no sequence $s_0, s_1, s_2, \ldots$ such that $s_i R s_{i+1}$ for all i). Prove that

$$\Box(\Box\varphi \to \varphi) \to \Box\varphi$$

is valid in $\mathcal{F}$ for all formulas φ.