Math 260AB - Introduction to Mathematical Logic

Winter and Spring 2012

Instructor: Sam Buss

Homework VI. Due: Monday, April 23

General instructions for all homeworks: Proofs are not required unless the problem asks for a proof.

Definition H is the halting problem. $\overline{H}$ is the complement of H.

1. Prove that $\overline{H}$ is many-one complete for the co-r.e. sets.

2. Prove that if R is r.e. and $Q \leq_m R$, then Q is r.e.

3. Prove that there is no many-one reduction from $\overline{H}$ to H. How would you modify your proof to show there is no many-one reduction from H to $\overline{H}$?

4. Suppose $R, Q \subseteq \Sigma^*$ and that $R \cap Q = \emptyset$. We say that R and Q are *recursively inseparable* if there is no recursive set S such that $R \subseteq S$ and $Q \subseteq \overline{S}$. Let

$$\begin{aligned} R &= \{\ulcorner M \urcorner : M \text{ accepts } \epsilon\}. \\ Q &= \{\ulcorner M \urcorner : M \text{ rejects } \epsilon\}. \end{aligned}$$

("ϵ" means the empty string, i.e., the blank tape.)

Prove that R and S are recursively inseparable.