General instructions for all homeworks: Proofs are not required unless the problem asks for a proof.

**Definition** $H$ is the halting problem. $\overline{H}$ is the complement of $H$.

1. Prove that $\overline{H}$ is many-one complete for the co-r.e. sets.

2. Prove that if $R$ is r.e. and $Q \leq_m R$, then $Q$ is r.e.

3. Prove that there is no many-one reduction from $\overline{H}$ to $H$. How would you modify your proof to show there is no many-one reduction from $H$ to $\overline{H}$?

4. Suppose $R, Q \subseteq \Sigma^*$ and that $R \cap Q = \emptyset$. We say that $R$ and $Q$ are *recursively inseparable* if there is no recursive set $S$ such that $R \subseteq S$ and $Q \subseteq \overline{S}$. Let

   $$ R = \{ \gamma M^\gamma : M \text{ accepts } \epsilon \} , $$

   $$ Q = \{ \gamma M^\gamma : M \text{ rejects } \epsilon \} . $$

   (“$\epsilon$” means the empty string, i.e., the blank tape.)

   Prove that $R$ and $S$ are recursively inseparable.