FINAL: MON 11:30-2:30
CENTR 216

HW10: Due Today

Useful problems to look at

* Problems from Midterm 1 and Midterm 2

* Simplify $\ln\left(\frac{(\tan x)\, x^x}{(x^3-1)\, e^{2x}}\right)$

* Find max/min and inflection points of
$g(x) = x^2 e^{-2x}$

* HW from 2.4, 4.5, 4.7, #82 on p.161

* Choose a and b to make f differentiable at $x=1$

$$f(x) = \begin{cases} \dfrac{x^2 e^x}{e} & \text{if } x \le 1 \\ x^3 + ax + b & \text{if } x > 1 \end{cases}$$

Exponential rules: (note e^x is always positive)

$e^0 = 1 \qquad e^a \cdot e^b = e^{a+b} \qquad (e^a)^b = e^{ab} \qquad \frac{e^a}{e^b} = e^{a-b}$

Logarithms:

$\ln 1 = 0 \qquad \ln(a \cdot b) = \ln a + \ln b \qquad \ln a^b = b \ln a$

$\ln\left(\frac{a}{b}\right) = \ln a - \ln b$

$\ln(e^x) = x \qquad e^{\ln x} = x$

Inverses:

$f^{-1}(x) = y$ is equivalent to $x = f(y)$

Shifting: starting with graph of $g(x)$

$g(x+k)$ ~ shift LEFT by k

$g(x-k)$ ~ shift RIGHT by k

$g(x)+k$ – shift vertically by k

$g(kx)$ ~ stretch horizontally by $\frac{1}{k}$

$k\,g(x)$ ~ stretch vertically by k

Even/Odd:

Even function: $f(-x) = f(x)$ (i.e. $x^2, \cos x$)

Odd function: $f(-x) = -f(x)$ (i.e. $x^3, \sin x$)

Trigonometry:

$\cos^2\theta + \sin^2\theta = 1$ $\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\sec\theta = \frac{1}{\cos\theta}$

$\sin(0) = 0$ $\cos(0) = 1$ $\tan(0) = 0$ $\sec(0) = 1$

Continuous: (no holes, jumps or asymptotes)

For f to be continuous at c

$$\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x) = f(c)$$

Derivatives:

Derivative = Instantaneous rate of change = slope of tangent line

$$\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

To evaluate derivative using the definition

(1) simplify expression $\frac{f(x+h) - f(x)}{h}$

(2) cancel h's

(3) evaluate at $h = 0$ (if you get $\frac{0}{0}$ still need to do work!!)

$(x+h)^2 = x^2 + 2xh + h^2$

$\frac{1}{a} - \frac{1}{b} = \frac{b-a}{ab}$

$$\text{units of } f'(x) = \frac{\text{units of } f(x)}{\text{units of } x}$$

Tangent line: (at $x=a$)

$$y = \underbrace{f'(a)}(x-a) + f(a)$$

this should be a number, not a function!!

Avoid sharp bends: (i.e. like $|x|$)

If $\lim_{x\to c^-} f'(x) = a$ and $\lim_{x\to c^+} f'(x) = b$

then to be differentiable at $x=c$ we need to (1) be continuous and

(2) have $a=b$ (to avoid sharp bend)

Interpreting derivatives:

$f' > 0 \longleftrightarrow f$ increasing

$f' < 0 \longleftrightarrow f$ decreasing

$f'' > 0 \longleftrightarrow f$ concave up

$f'' < 0 \longleftrightarrow f$ concave down

Rules for differentiation:

$\frac{d}{dx}(c\,f(x)) = c\,f'(x)$ $\quad$ $\frac{d}{dx}(f(x)+g(x)) = f'(x)+g'(x)$

$\frac{d}{dx}(f(x)g(x)) = f'(x)g(x) + f(x)g'(x)$

$\frac{d}{dx}\left(\frac{1}{f(x)}\right) = \frac{-f'(x)}{(f(x))^2}$ $\quad$ $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$

$\frac{d}{dx}(f(g(x))) = f'(g(x))\,g'(x)$

$\frac{d}{dx}(c)=0$ $\quad$ $\frac{d}{dx}(x^n) = n x^{n-1}$ $\quad$ $\frac{d}{dx}(e^x) = e^x$

$\frac{d}{dx}(\sin x) = \cos x$ $\quad$ $\frac{d}{dx}(\cos x) = -\sin x$ $\quad$ $\frac{d}{dx}(\tan x) = \sec^2 x$

$\frac{d}{dx}(\sec x) = \sec x \tan x$ $\quad$ $\frac{d}{dx}(\ln x) = \frac{1}{x}$

When differentiating:

* First see if we can "simplify" the function
* think about what rules to use
* make sure to get all the terms

Implicit differentiation:

(don't forget $\frac{dy}{dx}$)

* Differentiate both sides with respect to x
* Rearrange and solve for $\frac{dy}{dx}$

Min/Max:

p local min if $f(p) \le f(x)$ for x "near" p

p local max if $f(x) \le f(p)$ for x "near" p

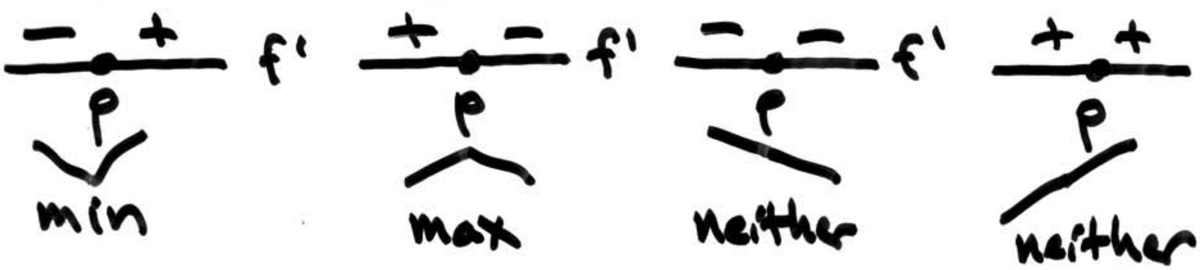
p global min if $f(p) \le f(x)$ for all x

p global max if $f(x) \le f(p)$ for all x

Critical Points = $f'(p) = 0$ or $f'(p)$ undefined

Min's and max's happen at critical pts

First derivative test: (p a critical pt)



Second derivative test:

$f'(p) = 0$ and $f''(p) > 0 \implies$ min

$f'(p) = 0$ and $f''(p) < 0 \implies$ max

$f'(p) = 0$ and $f''(p) = 0 \implies$ inconclusive, use first derivative test

Inflection Point = change of concavity

happens when $f''(p) = 0$ or $f''(p)$ undefined

+ − f'' or − + f''

Optimization problems:

(1) Understand the problem
- What are we looking for?
- What can we adjust?

(2) Draw a picture, label variables
- understand connections

(3) Set up a function which we can optimize to find the answer
- reduce down to one variable

(4) Take derivative, find critical points, test for min/max

(5) Having found min/max interpret the answer to give final result.

(*) Does your answer seem reasonable??

L'Hopital

If $\lim_{x \to a} \frac{f(x)}{g(x)} \to \frac{0}{0}$ or $\frac{\infty}{\infty}$

then $\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$

* Note: may need to use the rule several times