

Math 130A – Prof. Rabin – Last Homework Assignment – due December 6, 2007

(1) Compute e^{At} for each of the following matrices A :

(a)

$$\begin{bmatrix} 2 & 2 \\ 2 & -1 \end{bmatrix}.$$

(b)

$$\begin{bmatrix} 1 & 2 \\ -5 & -1 \end{bmatrix}.$$

(c)

$$\begin{bmatrix} 2 & 1 \\ 0 & 2 + \epsilon \end{bmatrix}.$$

In this last case, also find the limit of e^{At} as $\epsilon \rightarrow 0$. This is the sort of limit discussed in class, in which the two distinct eigenvalues of the matrix At come together, as do their eigenvectors. The result will be the exponential of the matrix obtained by setting $\epsilon = 0$, which is not diagonalizable.

(2) Solve the inhomogeneous linear system

$$\dot{\mathbf{x}} = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix} \mathbf{x} + \begin{bmatrix} e^{2t} \\ te^{2t} \end{bmatrix},$$

with the initial condition $\mathbf{x}(0) = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$.

(3) Do the following problems from the textbook:

6.1.3, 6.1.4, 6.1.6, 6.3.3, 6.3.6, 6.3.10.