

* $\dot{x} = Ax$, general form of solution is $\dot{x} = c e^{\lambda_1 t} x_1 + d e^{\lambda_2 t} x_2$ (similar for 3×3)
 $\lambda_1, \lambda_2 =$ evalues and x_1, x_2 corresponding eectors
 if A deficient (i.e. repeated eigenvalue and not enough eignevectors)

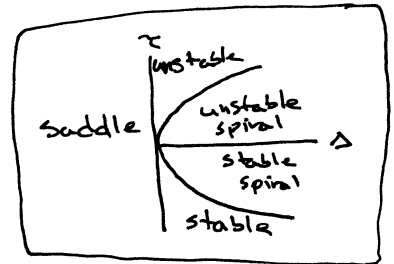
$$\dot{x} = c e^{\lambda t} x_1 + d e^{\lambda t} (t x_1 + x_2)$$

$x_1 =$ evector and x_2 satisfies $(A - \lambda I)x_2 = x_1$

if eigenvalues / eignevectors are complex to get "real" form take
 real part ($e^{\text{Re}(\lambda t)} x_1$) and imaginary part ($e^{\text{Im}(\lambda t)} x_1$)

* Stability

Complex eigenvalues $\left\{ \begin{array}{l} \text{real part} < 0 \rightarrow \text{stable spiral} \\ \text{real part} = 0 \rightarrow \text{ellipses} \\ \text{real part} > 0 \rightarrow \text{unstable spiral} \end{array} \right.$



zero an eigenvalue \rightarrow non-isolated fixed pts

two positive evalues \rightarrow unstable
 two negative evalues \rightarrow stable
 different signs \rightarrow saddle
 repeated evalue \rightarrow degenerate

When plotting distinguish between "fast" direction and "slow" direction

* Plotting

- (1) if complex determine if it spirals in/out and clockwise / counterclockwise
 \uparrow can be determined by how trajectory crosses x-axis
- (2) if real plot eignevector directions and mark in/out then plot other trajectories

* $\dot{x} = f(x, y)$
 $\dot{y} = g(x, y)$
 Fixed point: solve $\begin{cases} f(x, y) = 0 \\ g(x, y) = 0 \end{cases}$ (might be multiple solutions)

around fixed point (x^*, y^*) locally behaves like linearization

nullclines:

$f(x, y) = 0$ (soln. crosses vertically) $\left[\begin{array}{cc} \frac{\partial f}{\partial x}(x^*, y^*) & \frac{\partial f}{\partial y}(x^*, y^*) \\ \frac{\partial g}{\partial x}(x^*, y^*) & \frac{\partial g}{\partial y}(x^*, y^*) \end{array} \right]$
 $g(x, y) = 0$ (soln. crosses horizontally)

TO PLOT

- (1) Find fixed pts and nullclines, graph these
- (2) mark directions at nullclines and other regions
- (3) sketch solutions

General test tips:

- (1) Carefully read the questions, make sure to understand what is being asked before you answer, answer everything that is asked and don't do anything that is not asked
- (2) Carefully check your answers (do they make sense? can you check it?)
 triple check algebra, arithmetic and copying [leave time to do this]
- (3) Do the "easiest" problems first
- (4) If the calculations are getting messy (i.e. weird fractions, big square roots) then go back and double check arithmetic, very likely a mistake was made