Math 130B – Prof. Rabin – Homework Assignment 6 – due February 19, 2008

(1) Consider the system (a chemical reaction model)

$$\dot{x} = (1/8) - x + x^2 y, \qquad \dot{y} = (1/2) - x^2 y.$$

Show that the set $\{(x, y) : x \geq 1/16,\ 0 \leq y \leq 128,\ x + y \leq 130\}$ is positively invariant and must therefore contain a closed orbit.

(2) Consider the system (a more complicated predator-prey model)

$$\dot{x} = x\left(1 - \frac{x}{30} - \frac{y}{x+10}\right), \qquad \dot{y} = y\left(\frac{x}{x+10} - \frac{1}{3}\right).$$

(a) Find and classify the three fixed points.
(b) Show that the region R defined by

$$R = \{(x, y) : x \geq 0,\ y \geq 0,\ x + y \leq 50\}$$

is positively invariant.
(c) Show that for any point P in the interior of R (that is, in R but not on the lines forming its boundary), $\omega(P)$ is a periodic orbit.

(3) The terminology suggests that an "ω-limit point" should be a special type of "limit point". However, we have not yet compared the definitions carefully to verify this.
(a) Is it true that every ω-limit point of a trajectory $\mathbf{x}(t)$ is also a limit point of that trajectory?
(b) Can there be limit points of a trajectory that are not also ω-limit points? If so, give some examples.

(4) Do problem 7.3.11 from the textbook.