Supercurrents and Minimal Manifolds

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Abstract

Originally positive closed supercurrents were constructed as objects associated to tropical varieties, much the same way as ordinary closed positive currents are associated to complex subvarieties. We will show how one can also associate a supercurrent to any smooth submanifold of $\mathbb{R}^n$. The construction depends on the choice of a scalar product on $\mathbb{R}^n$, and the current reflects the induced Riemannian structure on the submanifold. This permits us to build a calculus on the submanifold analogous to the Kahler formalism on complex manifolds. The supercurrents that we define are not closed (except when the submanifold is linear), but it turns out the currents associated to minimal submanifolds satisfy another condition which is almost as strong. From there one can deduce e.g. area estimates for minimal surfaces along the lines of the classical results of Lelong and others.