HW1 Solution of Math170A

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1.1.25

Proof. Let $C^{(j)}$ be the jth column of matrix A, then

$$A = [C^{(1)} \cdots C^{(n)}]. \tag{1}$$

Now, consider the matrix multiplication in the following form

$$b = Ax = x_1 C^{(1)} + \cdots + x_n C^{(n)}, \tag{2}$$

which is a linear combination of A. □

1.2.4

Proof. Use contradiction, assume there is a nonzero y such that $Ay = 0$. Multiply A^{-1} on both sides, one has $y = A^{-1} \cdot 0 = 0$ which contradicts with $y \neq 0$. □

1.3.4

- Step 1
  $y_1 = 1$.
- Step 2
  $y_2 = \frac{3-(-1y_1)}{2} = 2$.
- Step 3
  $y_3 = \frac{2-3y_1-y_2}{-1} = 3$.
- Step 4
  $y_4 = \frac{9-4y_1-y_2-(-3y_3)}{3} = 4$.

Thus, $Y = [1\ 2\ 3\ 4]^T$.

1.4.15

Proof. (a). For any $x \in \mathbb{R}^2$ and $x \neq 0$, consider $x^T Ax = 4x_1^2 + 9x_2^{2>0}$, thus A is definite positive.

(b). Apply the method on the book, $R = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$.

(c). For those diag elements of R, wo do not force them to be positive sqare root, then the other three upper trianglular matrices can be derived

$$R_2 = \begin{bmatrix} -2 & 0 \\ 0 & 3 \end{bmatrix}. \quad R_3 = \begin{bmatrix} -2 & 0 \\ 0 & -3 \end{bmatrix}. \quad R_4 = \begin{bmatrix} 2 & 0 \\ 0 & -3 \end{bmatrix}. \tag{3}$$

(d). Since for each r_{ii}, once it is determined, the ith row of R is uniquely determined. Thus, for an n by n matrix, there are two options for each row of matrix factor R, so 2^n possiblities totally.

□

1.5.9

Proof. Assume the width $s = k$ where $k << n$. Then, firstly consider the first k steps for solving from x_n to x_{n-k+1}, we have

$$x_i = (y_i - \sum_{j=i+1}^{n} r_{i,j}x_j)/r_{i,i}, \quad n-k+1 \leq i \leq n. \tag{4}$$

Thus, the flops of first k steps is $f_1 = 1 + 3 + \cdots + (2k-1) = k^2$.

Next, consider the rest $n-k$ steps for solving from x_{n-k} to x_1, since all $r_{i,j} = 0$ when $j - i > k$, we have

$$x_i = (y_i - \sum_{j=i+1}^{i+k} r_{i,j}x_j)/r_{i,i}, \quad 1 \leq i \leq n-k. \tag{5}$$

Thus, the flops of the rest steps is $f_2 = (n-k) \cdot (2k+1)$.

Finally, the total flops is $f(n) = f_1 + f_2 = (n-k)(2k+1) + k^2$. Consider $k << n$, $k^2 \approx 0$, and therefore $f(n) \approx (n-k)(2k+1) \approx 2nk$.

□

1.7.10

(a). Use matlab command det() to find the determiants of four submatrice.

(b). Apply the Guassian elimination for matrix A, the result upper triangular matrix

$$U = \begin{bmatrix} -6 & -1 & 2 & -3 \\ 0 & 2/3 & -1/3 & 2 \\ 0 & 0 & -1/2 & 3 \\ 0 & 0 & 0 & -3 \end{bmatrix}. \tag{6}$$

and

$$y = \begin{bmatrix} -14 \\ 8(1/3) \\ 10(1/2) \\ -12 \end{bmatrix}. \tag{7}$$

(c). Refer to 1.3.4, the solution is $x = [1\ 2\ 3\ 4]^T$.