

CONFERENCE ON STARK'S CONJECTURES AND RELATED  
TOPICS – U. C. SAN DIEGO, SEPTEMBER 20–22, 2013

TITLES AND ABSTRACTS

ALINA BUCUR

**Title:** Statistics for zeta zeros of curves over finite fields

**Abstract:** We discuss in this talk statistics for the zeroes of zeta functions of curves over finite fields. After the general case, we will concentrate on the family of Artin–Schreier covers of the projective line. Because of the special role played by the characteristic of the base field in the definition of Artin–Schreier curves, one can write their zeta functions in terms of exponential sums. This was first used by Entin to study the variation of the zeroes and the powers of the zeroes for the Artin–Schreier curves of  $p$ -rank 0. Other statistics were obtained for the same family by A. Bucur, C. David, B. Feigon, M. Lalin and K. Sinha. More recently, similar statistics were obtained for other families of Artin–Schreier curves corresponding to different strata of the  $p$ -rank stratification. We show in this talk that for all families considered, the distribution of the (properly normalized) zeroes of the zeta function in intervals follows a Gaussian distribution. This is joint work with C. David, B. Feigon and M. Lalin.

TED CHINBURG

**Title:** Higher Chern classes in Iwasawa Theory

**Abstract:** In this talk I will discuss how various Main Conjectures in Iwasawa theory can be interpreted as statements about the first Chern classes of Iwasawa modules and complexes. I'll then discuss Main Conjectures for higher Chern classes. This is joint work with F. Bleher, G. Pappas, R. Greenberg, M. Kakde, R. Sharifi and M. Taylor.

HENRI DARMON

**Title:**  $p$ -adic iterated integrals, modular forms of weight one, and Stark-Heegner points.

**Abstract:** I will formulate a conjecture on the values of certain “ $p$ -adic iterated integrals” attached to a triple  $(f, g, h)$  of cusp forms of weights  $(2, 1, 1)$ , involving (in the case where  $f$  is attached to an elliptic curve) the  $p$ -adic logarithms of certain algebraic points on this curve. Relations with the Euler systems of Kato-Perrin-Riou, Beilinson-Flach, and Gross-Schoen will be evoked, as well as connections with

Stark units and Stark-Heegner points. This is an account of work in progress in collaboration with Alan Lauder and Victor Rotger.

### SAMIT DASGUPTA

**Title:** On the principal minors of the Gross Regulator

**Abstract:** In 1981, Gross stated a conjecture relating the leading term of the  $p$ -adic  $L$ -function of a character of a totally real field to a  $p$ -adic regulator of  $p$ -units in an abelian extension of the field. We will describe a refinement of the conjecture that gives a formula for the principal minors of Gross's regulator matrix. In the specific case of the diagonal elements of the matrix, we recover a conjectural formula for these units that I proposed earlier. This is joint work with Michael Spiess.

### ADRIAN DIACONU

**Title:** On Higher Moments of Quadratic Dirichlet  $L$ -Functions

**Abstract:** In this talk we give a cohomological description of the " $p$ -parts" of the multiple Dirichlet series attached to moments of quadratic  $L$ -series over a number field. We expect that the Eisenstein Conjecture of Bump, Brubaker and Friedberg extends to the relevant Kac-Moody groups in our situation, and that the multiple Dirichlet series we construct occurs (perhaps, up to a normalization) in a *Fourier-Whittaker* coefficient of a (Kac-Moody) Eisenstein series.

### WILLIAM DUKE

**Title:** Harmonic Maass forms of weight one

**Abstract:** I will describe work with Yingkun Li on some arithmetic properties of the Fourier coefficients of harmonic modular forms of weight one. These are Maass forms of weight one whose eigenvalue under the Laplacian is zero and that are allowed to have polar-type singularities in the cusps. We show that in some dihedral cases they are connected to the Galois representations associated to newforms of weight one in that their holomorphic Fourier coefficients are logs of algebraic numbers in the field where the Galois representation lives. We also find numerical evidence for this in several exotic cases. These Fourier coefficients seem to behave like logs of derivatives of special values of  $L$ -functions.

### SOLOMON FRIEDBERG

**Title:** Eisenstein series, Gauss Sums and Crystal Graphs

**Abstract:** The Whittaker coefficients of Eisenstein series on covers of groups are Dirichlet series that have analytic continuation and functional equation. However they are not Langlands  $L$ -functions, in fact they are not Eulerian. Nonetheless their coefficients are algebraic and may be expressed in terms of higher order Gauss

sums. This description is governed by representation theory, and more precisely by paths on crystal graphs. We shall report on this; it is joint work with Brubaker and Bump (type A) and Zhang (types B,C, with other the calculations for other types on-going).

### JEFFREY HOFFSTEIN

**Title:** Second Moments and simultaneous non-vanishing of  $GL(2)$  automorphic  $L$ -series

**Abstract:** I'll obtain a second moment formula for the  $L$ -series of holomorphic cusp forms, averaged over twists by Dirichlet characters modulo a fixed conductor  $Q$ . The estimate obtained has no restrictions on  $Q$ , with an error term that has a close to optimal power savings in the exponent. However, one of the contributions to the main term is a special value of a shifted double Dirichlet series. I'll show that this special value is small on average, and obtain a corresponding estimate for a mean value of the second moment over  $\mathbb{Q}$ . This mean value is non-zero even when applied to a product of two distinct  $L$ -series, leading to a simultaneous non-vanishing result. The approach uses the theory of shifted multiple Dirichlet series to obtain some refined estimates for double shifted sums. These estimates are the key ingredient in the second moment estimate.

### JEFFREY LAGARIAS

**Title:** The Lerch zeta function and related functions

**Abstract:** The Lerch zeta function is a three-variable generalization of the Riemann zeta function. This talk surveys results on algebraic and analytic structures associated to variants of this function. These structures include a multivalued analytic continuation of this function in three complex variables, and its relation to on functional equations and differential equations satisfied by this function. We introduce two-variable Hecke operators for which the Lerch zeta function is an eigenfunction. We illustrate a parallel of these operators with Kubert operators, as studied by J. Milnor. These results were obtained in joint work with W.-C. Winnie Li.

### RAM MURTY

**Title:** Special values of Stark's series

**Abstract:** If  $Q$  is a positive definite binary quadratic form with integer coefficients and  $\chi$  is a Dirichlet character, Stark defined the  $L$ -function

$$L_Q(s, \chi) = \sum_{(m,n) \neq (0,0)} \frac{\chi(Q(m,n))}{Q(m,n)^{-s}}.$$

We will discuss the transcendence of the special value of this  $L$ -series at  $s = 1$  and at other points. This is joint work with V. Kumar Murty.

**CRISTIAN D. POPESCU**

**Title:** The Brumer–Stark conjecture and applications

**Abstract:** I will discuss recent results towards the Brumer–Stark conjecture (joint with Cornelius Greither) and some of their applications to the study of higher even Quillen  $K$ –groups of rings of algebraic integers of CM and totally real number fields (joint with Grzegorz Banaszak.)

**KENNETH RIBET**

**Title:** Eisenstein ideals at non-prime levels

**Abstract:** Barry Mazur’s 1977 landmark “Eisenstein ideal” article provided a definitive discussion of Eisenstein-like behavior for the space of weight-two cusp forms on  $\Gamma_0(N)$  where  $N$  is a prime number. What if the weight is no longer 2? What if the level is no longer prime? Faltings–Jordan taught us that transition to weights  $\geq 2$  is relatively smooth: one needs to think in terms of crystalline representations rather than representations associated with finite flat group schemes. For non-prime level, the situation is currently far from clear. I will focus on the case where the level is a product of two distinct primes and explain what seems to have been established to date. In part, I will rely on the 2013 PhD thesis of H. Yoo, who focused especially on “multiplicity” (i.e., dimension) questions.

**KARL RUBIN**

**Title:** Refined class number formulas in the higher rank case

**Abstract:** In this talk we present a generalization of Darmon’s 1995 “refined class number formula” for cyclotomic units. The generalization relates Stark units over different number fields. Using Kolyvagin systems we can prove the conjecture in certain cases. This is joint work with Barry Mazur.

**HAROLD STARK**

**Title:** Modular forms and Poincaré series

**Abstract:** TBA

**DANIEL VALLIÈRES**

**Title:** The abelian Stark conjecture without split places

**Abstract:** We will explain a generalization of the Rubin–Stark conjecture where the set  $S$  of places involved does not necessarily contain  $r$  split places anymore. One replaces this previous condition with the one that all imprimitive  $L$ –functions have order of vanishing at least  $r$  at zero.